Generalized P Colony Automata

Multiset Processing with Systems of Simple Components with Complex Behavior

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- **Multisets: collection** of objects/symbols, **multiplicities**

- **Complex behavior:** computational completeness, universality

- **Simple building blocks:** simple symbol processing **agents** in a **shared environment** (multiset) which they modify

Chemical metaphor

- A "**chemical style**" approach to the notion of **computation**

data	substances or molecules
processing	chemical reaction
algorithm	substances and their reaction laws

- **Data structure:** multisets
- **Computation:** multiset transformation/processing

Outline

- P colonies
  - structure, functioning, computational power, **multiset languages**
- P colony automata
  - languages of **strings of symbols**
- Generalized P colony automata
  - languages of **strings/sequences of multisets**

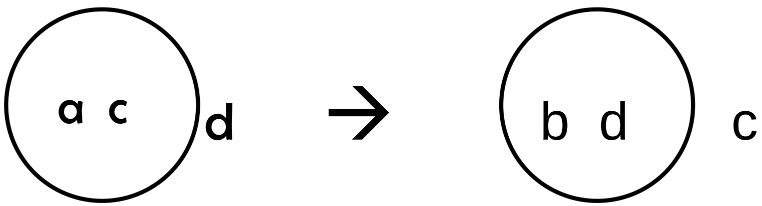
P colonies

- A population of very **simple cells** in a **shared environment**:
  - **Fixed number** of objects (1, 2, 3) inside each cell
  - **Simple** rules (programs) for **moving** and **changing** the objects

- The objects are **exchanged** directly only between the **cells** and the **environment**

[Kelemen, Kelemenova, Paun 2004]

P colonies



rewriting + communication

$$(a \to b, c \leftrightarrow d)$$

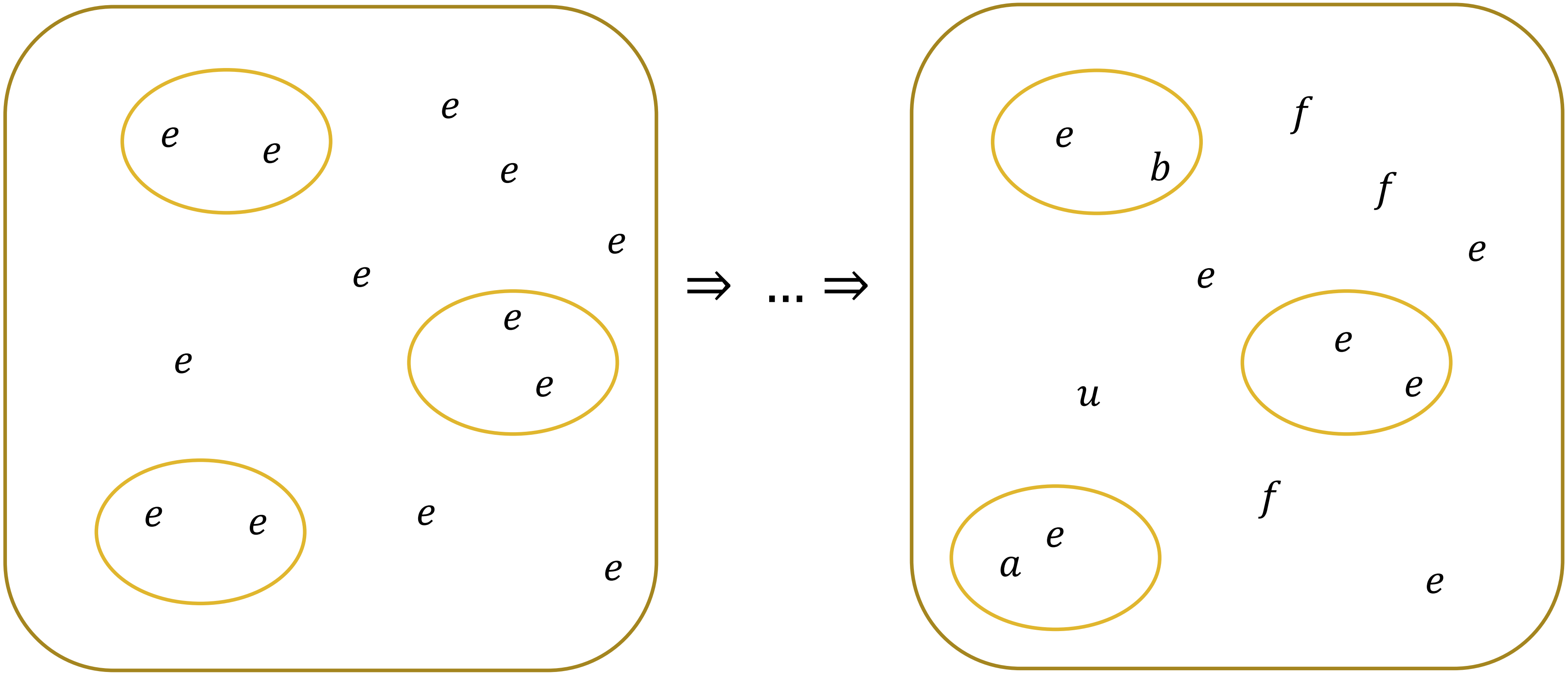
The computation

- Start in an **initial configuration**
- Apply the programs (sets of rules) in **parallel** in the cells, **halt** if no program is applicable
- The **result** is the **number** of the **multiplicities** of certain objects found in the **environment**

The computation

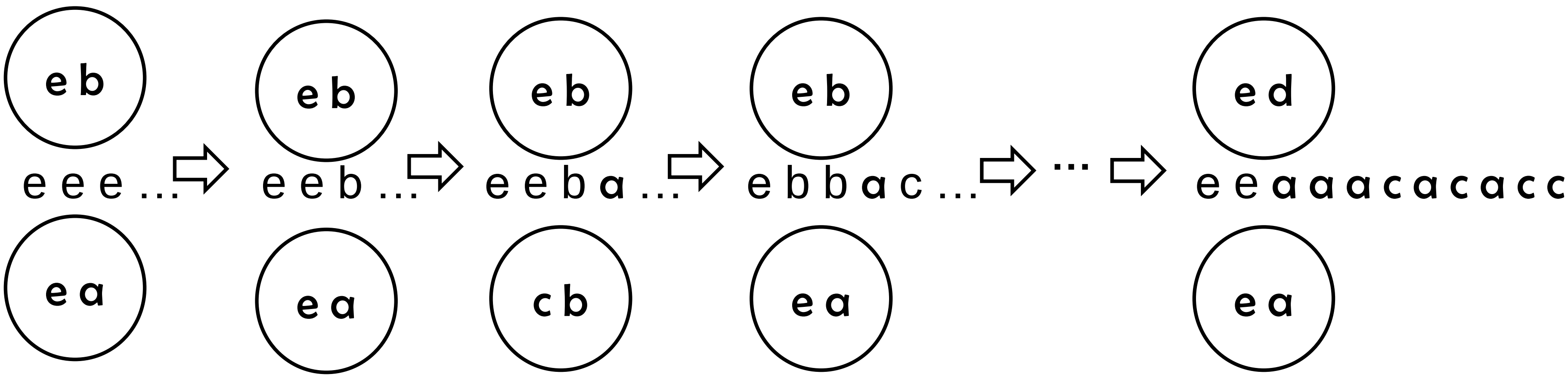
initial configuration

a possible result



The computation

$(e \rightarrow b, b \leftrightarrow e)$
$(e \rightarrow d, b \leftrightarrow e)$



$(e \rightarrow c, a \leftrightarrow b)$
$(b \rightarrow a, c \leftrightarrow e)$

We obtain $a^n c^n, n \geq 1$ in the environment.

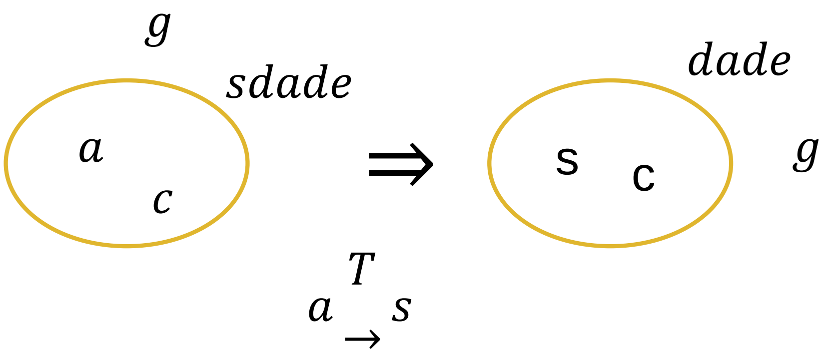
P colony automata

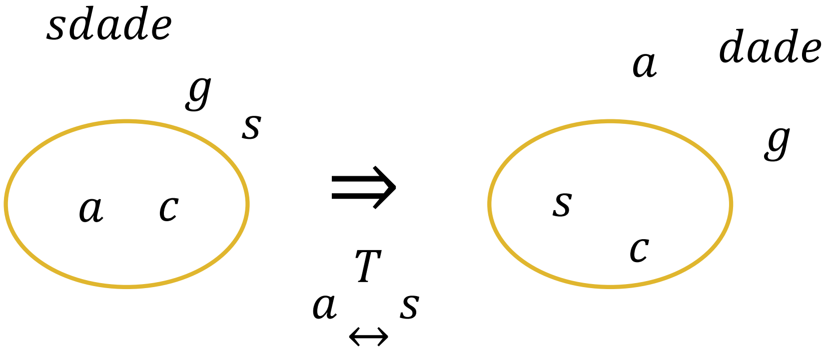
- Response to the **changes in the environment**
- **Automata-like** behavior - an **input string** is given
- **Tape rules** and **non-tape rules**: the application of programs with tape rules **reads a symbol** of the input

[Ciencialova, Cienciala, Csuhaj-Varjú, Kelemenova, Vaszil 2010]

P colony automata

The effect of tape rules:





Different computational modes...

...with different uses of the tape rules:

- *t-transition*, denoted by $\Rightarrow_t$, if $u' = u$ and P_c is maximal set of programs with respect to the property that every $p \in P_c$ is a tape program with $read(p) = a$;
- *tmin-transition*, denoted by $\Rightarrow_{tmin}$, if $u' = u$ and P_c is maximal set of programs with at least one $p \in P_c$, such that p is a tape program with $read(p) = a$;
- *tmax-transition*, denoted as $\Rightarrow_{tmax}$, if $u' = u$ and $P_c = P_T \cup P_N$ where P_T is a maximal set of applicable tape programs with $read(p) = a$ for all $p \in P_T$, the set P_N is a set of nontape programs, and $P_c = P_T \cup P_N$ is maximal;
- *n-transition*, denoted by $\Rightarrow_n$, if $u' = au$ and P_c is maximal set of nontape programs.

Power of the different modes

- **nt, ntmax, ntmin**: any recursively enumerable language can be accepted/characterized

[Ciencialova, Cienciala, Csuhaj-Varjú, Kelemenova, Vaszil 2010]

- **t, one cell**: only CS languages can be generated

[Cienciala, Ciencialova 2011a]

- **initial:** any recursively enumerable language can be characterized

[Cienciala, Ciencialova 2011b]

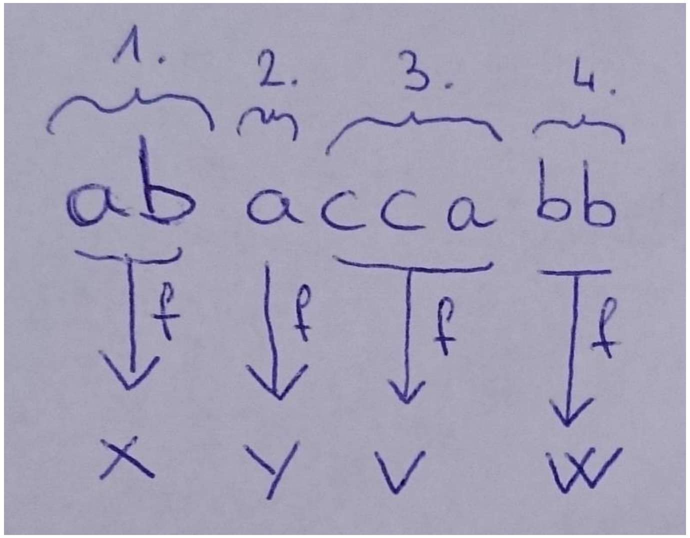
Generalized P colony automata

- A **maximal parallel set** of programs is chosen
- The tape rules might “read” **several different symbols (multiset) in one step**.
- The **set of input sequences accepted** by a GenPCol: The set of the sequences of read multisets

Generalized P colony automata

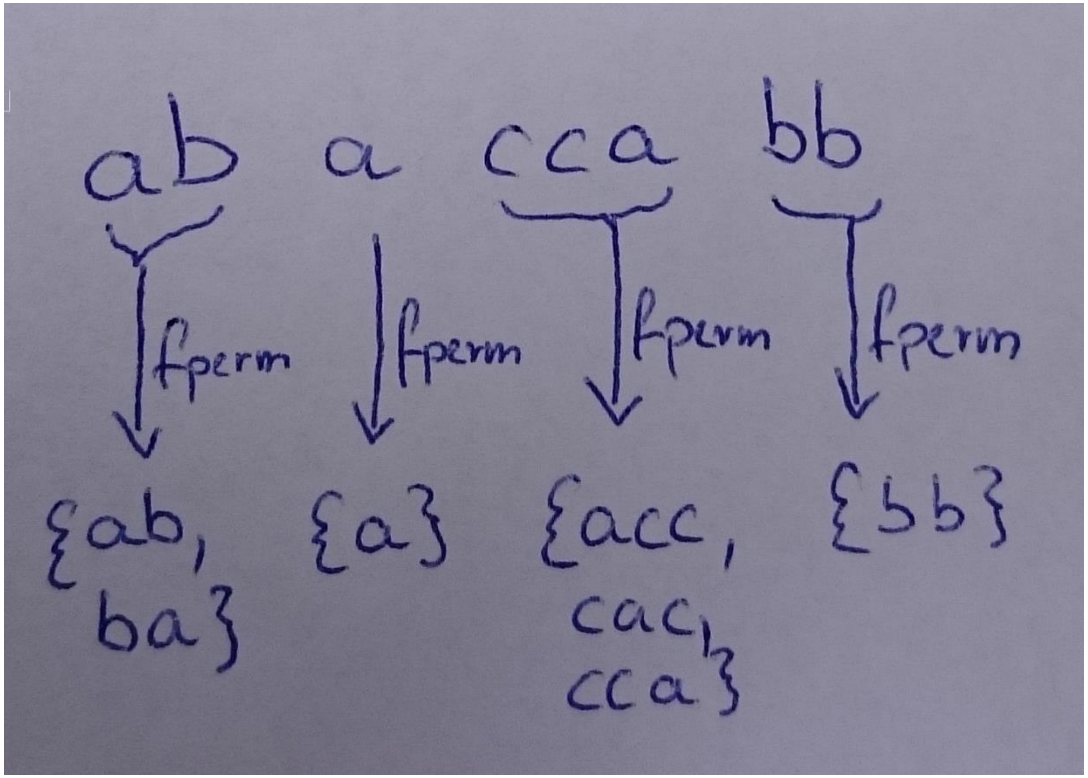
- The **language accepted** by a GenPCol in respect to a mapping ($f: (V - \{e\})^* \rightarrow 2^{\Sigma^*}$):

$$\mathcal{L}(\Pi, f) = \{f(u_1) \cdot \dots \cdot f(u_s) \in \Sigma^* \mid u_1 \dots u_s \text{ is an accepted input sequence}\}$$



f_{perm}

- $f_{perm}: (V - \{e\})^* \rightarrow 2^{(V-\{e\})^*}$, where $f(x) = \{y \in (V - \{e\})^* \mid y = perm(x)\}$



Generalized P colony automata modes using f_{perm}

- **All-tape**: all programs contain at least one tape rule
- **Com-tape**: all communication rules are tape rules
- **No restriction (noted by *)**

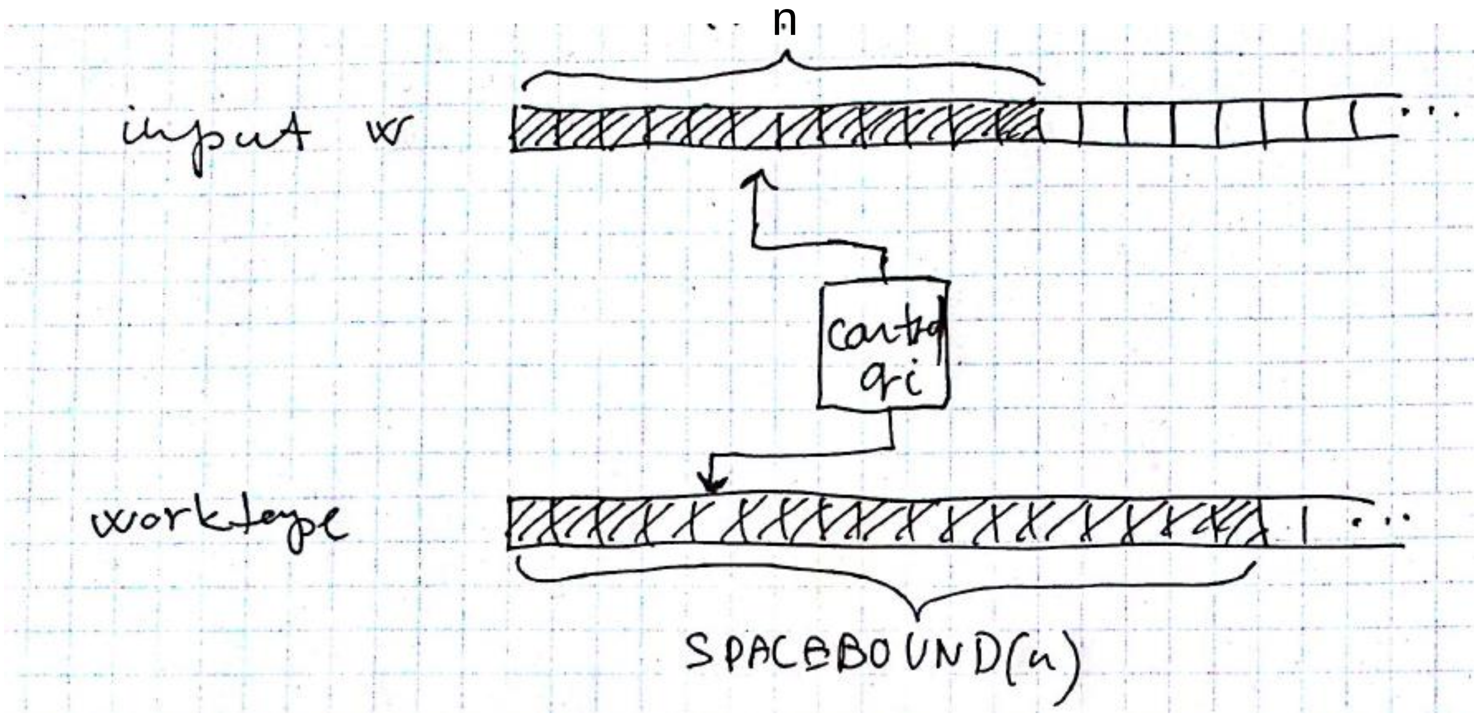
[Kántor, Vaszil 2013]

Turing machines with restricted space bound

A nondetermininstic Turing machine with a **one-way** input tape is **restricted** $S(n)$ **space bounded** if the number of **nonempty cells** on the worktape(s) is **bounded by** $S(d)$, where d is the **distance of the reading head** from the left-end of the one-way input tape.

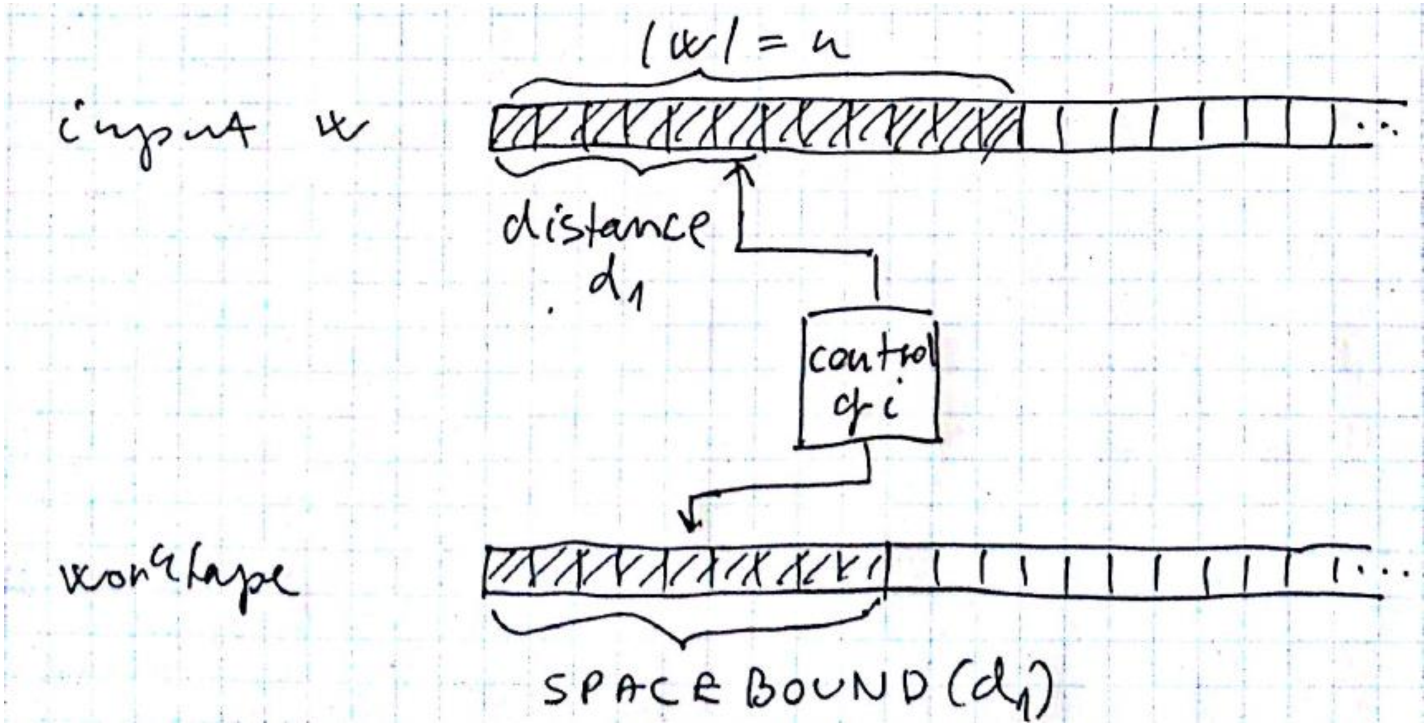
A Turing machine with SPACEBOUND(n)

The length of the available worktape is bounded by the length of the input:



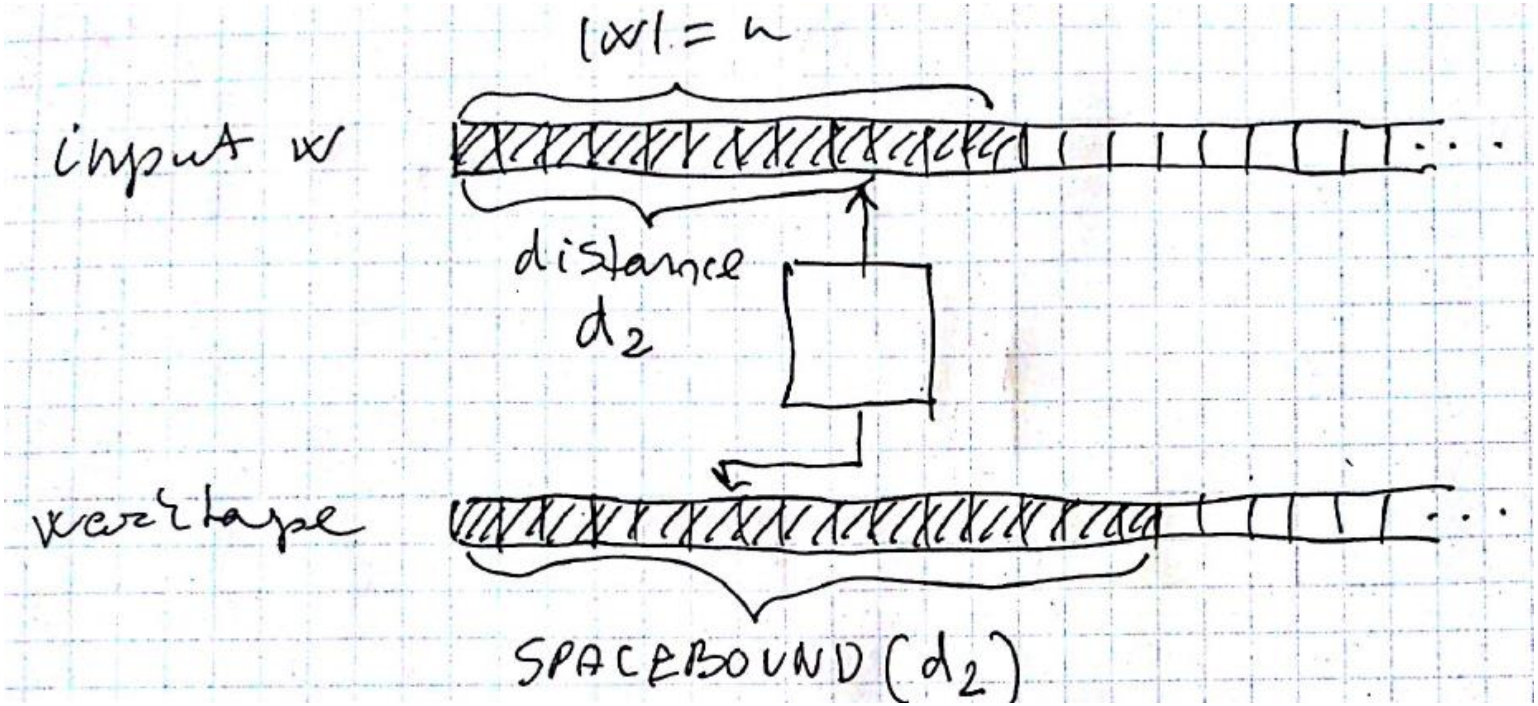
Turing machines with *restricted* space bound

1. After reading d_1 input cells:



Turing machines with *restricted* space bound

2. After reading d_2 input tape cells:



Computational power

- $\mathcal{L}(GenPCol, com-tape, f_{perm}) \cup \mathcal{L}(GenPCol, all-tape, f_{perm}) \subseteq \mathcal{L}(GenPCol, *, f_{perm})$

- $\mathcal{L}(GenPCol, com-tape, f_{perm}) \cap \mathcal{L}(GenPCol, all-tape, f_{perm}) - \mathcal{L}(CF) \neq \emptyset$

- $\mathcal{L}(REG) \subset \mathcal{L}(GenPCol, all-tape, f_{perm}) \cup \mathcal{L}(GenPCol, com-$

New results: prerequied knowledge

- $For\ every\ language\ L \subseteq V^*, L \in LP, which\ is\ not\ regular\ there\ is\ a\ string\ w \in L\ which\ can\ be\ written\ in\ the\ form\ w = w_1abw_2, for\ some\ w_1, w_2 \in\ V^*$ and $\mathrm{a}, \mathrm{b} \in V\ such\ that\ w_1baw_2 \in L$.

[Freund, Kogler, Paun, Pérez-Jiménez, 2009]

New results

- $\{(ab)^n(cd)^n \mid n \geq 1\}$ can be accpeted by a GenPCol in all-tape mode using f_{perm}.
- Proof: Let us consider the following GenPCol (1 cell, 2 capacity):

New results

$$\{x \xrightarrow{T} a, e \leftrightarrow e\},$$
$$\{e \xrightarrow{T} b, a \leftrightarrow e\},$$
$$\{b \xrightarrow{T} a, e \leftrightarrow e\},$$
$$\{b \xrightarrow{T} c, e \leftrightarrow e\},$$
$$\{c \xrightarrow{T} d, e \leftrightarrow a\},$$
$$\{d \leftrightarrow e, a \xrightarrow{T} c\}$$

New results, open problems summary

- Acceptance of $\{(ab)^n(cd)^n \mid n \geq 1\}$ by a GenPCol in all-tape mode using $f_{perm}(\Pi) \implies \Pi$ is able to accept a language that P automata with input mapping f_{perm} cannot
- Open question: $\mathcal{L}\big(GenPCol, all-tape, f_{perm}\big) \supset \mathcal{L}(PA, f_{perm})$?
- Open question: Power comparison of all-tape and com-tape modes?
- Open question: Computational power using other mapping functions?

Final slide

Thank you for your attention!