Membrane Computing Applications in Computational Economics

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Contents

1. Preliminaries
2. Producer – Retailer problem: Initial Model
   • Description.
   • Formalization.
   • Implementation in P – Lingua & MeCoSim.
   • Simulation & Results discussion.
   • Description.
   • Formalization.
   • Implementation in P – Lingua & MeCoSim.
   • Simulation & Results discussion.
4. Further developments
Motivation

• Success of MC modeling biological systems
• Translation to unexplored field: Economic Modeling
• Replication of Păun’s Producer - Retailer Problem results:
  • Selection of the proper type of P System
  • Economic processes modeling
  • Implementation in P-Lingua & MeCoSim
  • Simulate & discuss results
• Extension of the original model with new economic processes:
  • Identification and modeling of processes
  • Implementation & simulation
• Further developments
Why not extend to other fields?

- **Computational economics:**
  - Computational modeling of economic systems (ODEs, ABM, ...)

- **Up-to-date efforts:**
  - Polish authors: Korczynski (2005)
  - Păun’s efforts:
Păun’s proposals

- Encourage researchers of other areas to use P Systems.
- Suggests modeling of some processes:
  - Production of goods
  - Order of goods
  - Purchase transactions:
    - Preferences between pairs (producer, retailer)
    - Geographical barriers
    - No distinction between counterparts
  - Monetary unit exchange
  - Capacity increase
Producer – Retailer Problem

Membrane Computing Applications in Computational Economics
Model Entities

- **Actors:**
  - Producers:
    - \((b_i, u_i) \rightarrow \text{capacity, money}\)
  - Retailers
    - \((c_j, v_j) \rightarrow \text{capacity, money}\)

- **Generic sources:**
  - Of raw material \((u_S, \text{generation rate})\)
  - Of demand or Generic consumer \((u_C, \text{demand rate})\)
Model interactions

- **Good’s and order’s flows:**
  - Producers generate good $d$ from raw material.
  - Retailers receive order $\bar{d}$ from generic consumer.
  - $d$ and $\bar{d}$ are matched
  - Purchase $P_{i,j} = P(\text{producer } i, \text{retailer } j)$

- **Monetary flows:**
  - Monetary unit exchange ($u_S, u_C, u_i, u_j$)
  - A set of prices.

- **External monetary injection:**
  - Key role for system evolving.
Summarized actors & interactions
Păun’s proposed system dynamic

- Presents a system behavior simulation:
Proposal - drawbacks

- Păun sketches the model:
  - No indications about:
    - Type of P System to be used.
    - The sequence of steps of the cyclic behavior.
    - The competing set of rules to be used.
  - Probabilities associated to rules in a strange way.
  - Randomness introduced in a naive way.

\[
\begin{align*}
[d_1 \bar{d}_1 v_1^{\text{price}}]_2 & \xrightarrow{p_{11}} [b_1 c_1 u_1^{\text{price}}]_2 \\
[d_2 \bar{d}_1 v_1^{\text{price}}]_2 & \xrightarrow{p_{12}} [b_2 c_1 u_2^{\text{price}}]_2 \\
[d_3 \bar{d}_1 v_1^{\text{price}}]_2 & \xrightarrow{p_{13}} [b_3 c_1 u_3^{\text{price}}]_2
\end{align*}
\]

\[\sum_i p_{ij} = 1\] Non-as-usual
Reproducing Păun’s system evolution

- Define a so-called: Initial Model

- Steps:
  - Select a type of P System -> PDP System.
    - Probabilities associated to rules.
    - Success in ecosystem modeling.
  - Define the steps of the cycle:
    - Associated to the transactions.
  - Formalize the model.
  - Establish the set of rules:
    - Following Păun’s guidelines.
    - Avoid problems associated to “strange” probabilities.
Defining steps of cycle

1. Initialization
   - Aggregate demand creation
   - Raw material disposability

2. Production
   - Orders generation
   - Production of goods

3. Authorization
   - Purchase authorizations generation

4. Transaction
   - Purchase transactions

5. Cleaning
   - Cleaning and technical rules

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Membrane Computing Applications in Computational Economics
Model Formalization (I)

\[ \Pi = (G, \Gamma, \Sigma, T, \mathcal{R}_E, \mu, \mathcal{R}_\Pi, \{ f_r \in \mathcal{R}_\Pi \}, M_1, M_2) \]

PDP System of degree (2,1)

Where:

- \( G = (V, E) \) with \( V = \{ e_1 \} \) and \( E = \{(e_1, e_1)\} \).
- Working alphabet: \( \Gamma = \{ b_i, d_i, u_i, c_j, \bar{d}_j, v_j, \bar{e}_j, f_{i,j} : 1 \leq i \leq k_1, 1 \leq j \leq k_2 \} \cup \{ R_1, R_2 \} \cup \{ C, S, \bar{d}, a, u_C, u_S \} \)

Where:

- \( C \): aggregate generic consumer.
- \( S \): raw material supplier.
- \( \bar{d} \): unit of aggregate demand from \( C \).
- \( a \): unit of supplied raw material provided by \( S \).
- \( u_C \): monetary unit owned by \( C \).
- \( u_S \): monetary unit owned by \( S \).
- \( b_i \): unit of production capacity of producer \( i. 1 \leq i \leq k_1 \).
- \( d_i \): unit of good supplied by producer \( i. 1 \leq i \leq k_1 \).
- \( u_i \): monetary unit owned by producer \( i. 1 \leq i \leq k_1 \).
- \( c_j \): unit of capacity of retailer \( j. 1 \leq j \leq k_2 \).

\( \bar{d}_j \): unit of good demanded by retailer \( j. 1 \leq j \leq k_2 \).

\( v_j \): monetary unit owned by retailer \( j. 1 \leq j \leq k_2 \).

\( \bar{e}_j \): unit of good demanded by retailer and authorized for transaction unit of \( \bar{d}_j. 1 \leq j \leq k_2 \).

\( f_{i,j} \): authorization for \( \bar{d}_j \) to be exchange with \( d_i. 1 \leq i \leq k_1, 1 \leq j \leq k_2 \).

\( R_1, R_2 \): for technical reasons.
Model Formalization (II)

- $\Sigma = \emptyset$.
- $R_E = \emptyset$.
- $\Pi = \{\Gamma, \mu, M_1, M_2, R_\Pi\}$ where:
  - Membrane structure: $\mu = [\ [ ]_2 ]_1$.
  - $M_1 = \{C, S, R_1, R_2\} \cup \{b_{i}^{k_{i,1}}, u_{i}^{k_{i,2}} : 1 \leq i \leq k_1\} \cup \{c_{j}^{k_{j,3}} : 1 \leq j \leq k_2\}$

Initial multisets contain basically:

- $b_{i}^{k_{i,1}}, u_{i}^{k_{i,2}}$: producers’ initial parameters.
- $c_{j}^{k_{j,3}}$: retailers’ initial capacities.

Where:

- $k_{i,1}$: initial production capacity of producer $i$. $1 \leq i \leq k_1$.
- $k_{i,2}$: initial monetary units of producer $i$. $1 \leq i \leq k_1$.
- $k_{j,3}$: initial capacity of retailer $j$. $1 \leq j \leq k_2$. 
Model Parameters

Goal: maximize model parametrization

- $k_1$: total number of producers.
- $k_2$: total number of retailers.
- $k_3$: units of raw material inserted into the system by $S$.
- $k_4$: allowed deviation from $k_3$.
- $k_5$: units of aggregate demand inserted into the system by $C$.
- $k_6$: allowed deviation from $k_5$.
- $k_7$: price fixed by $S$ for each unit of a.
- $k_8$: price fixed by $C$ as an estimation of each order of good.
- $k_{i,1}$: initial production capacity of producer $i$. $1 \leq i \leq k_1$.
- $k_{i,2}$: initial monetary units of producer $i$. $1 \leq i \leq k_1$.
- $k_{j,3}$: initial capacity of retailer $j$. $1 \leq j \leq k_2$.
- $k_{m,4}$: discrete prob. distribution of units of raw material inserted into the system by $S$. $1 \leq m \leq 3$.
- $k_{m,5}$: discrete prob. distribution of units of aggregate demand inserted into the system by $C$. $1 \leq m \leq 3$.
- $k_{i,6}$: price fixed by producer $i$ for each unit of $d_i$. $1 \leq i \leq k_1$.
- $k_{j,7}$: price fixed by retailers $j$ for each order of good. $1 \leq j \leq k_2$. 
Set of rules – Initialization

- Step 1.a: raw material disposability

\[ r_1 \equiv R_1 s[\ldots]_2^{k_{1,4}} a^{k_3+k_4} s[ R_1 ]_2^+ \]
\[ r_2 \equiv R_1 s[\ldots]_2^{k_{2,4}} a^{k_3} s[ R_1 ]_2^+ \]
\[ r_3 \equiv R_1 s[\ldots]_2^{k_{3,4}} a^{k_3-k_4} s[ R_1 ]_2^+ \]
\[ r_4 \equiv R_1 s[\ldots]_2^{1-k_{1,4}-k_{2,4}-k_{3,4}} a^{k_3-2*k_4} s[ R_1 ]_2^+ \]

- Step 1.b: generic demand creation

\[ r_5 \equiv R_2 c[\ldots]_2^{k_{1,5}} \bar{d}^{k_5+k_6} u_C^{(k_5+k_6)*k_8} c[ R_2 ]_2^+ \]
\[ r_6 \equiv R_2 c[\ldots]_2^{k_{2,5}} \bar{d}^{k_5} u_C^{k_5*k_8} c[ R_2 ]_2^+ \]
\[ r_7 \equiv R_2 c[\ldots]_2^{k_{3,5}} \bar{d}^{k_5-k_6} u_C^{(k_5-k_6)*k_8} c[ R_2 ]_2^+ \]
\[ r_8 \equiv R_2 c[\ldots]_2^{1-k_{1,5}-k_{2,5}-k_{3,5}} \bar{d}^{k_5-2*k_6} u_C^{(k_5-2*k_6)*k_8} c[ R_2 ]_2^+ \]

- Katie’s initial units of raw material inserted into the system by \( S \).
- \( k_4 \): allowed deviation from \( k_3 \).
- \( k_{m,4} \): discrete prob. distr. of units of raw material inserted.
- \( k_5 \): units of aggregate demand inserted by \( C \).
- \( k_6 \): allowed deviation from \( k_5 \).
- \( k_{m,5} \): discrete prob. distr. of units of aggregate demand inserted.
Set of rules – Production

- **Step 2.a: producer operation**
  \[ r_9 \equiv a \cdot b_i \cdot u_i^{k_7} \cdot [ ]_2^+ \rightarrow u_S^{k_7} \cdot [ d_i ]_2^0 \quad 1 \leq i \leq k_1 \]

- **Step 2.b: retailer operation**
  \[ r_{10} \equiv \bar{d} \cdot c_j \cdot u_C^{k_{j,7}} \cdot [ ]_2^+ \rightarrow [ \bar{d} \cdot v_j^{k_{j,7}} ]_2^0 \quad 1 \leq j \leq k_2 \]

- \( k_1 \): total number of producers.
- \( k_2 \): total number of retailers.
- \( k_7 \): price fixed by \( S \) for each unit of \( a \).
- \( k_{j,7} \): price fixed by retailers \( j \) for each order of good.
Set of rules – Auth. & Trans.

- **Step 3: Purchase auth. generation**

  \[ r_{14} \equiv [\tilde{d}_1]_2 \rightarrow [\tilde{e}_1 f_{1,1}]_2 \]

  - Geo-barriers

  \[ r_{15} \equiv [\tilde{d}_1]_2 \rightarrow [\tilde{e}_1 f_{1,2}]_2 \]

  - Non-preferences

  \[ r_{16} \equiv [\tilde{d}_2]_2 \rightarrow [\tilde{e}_2 f_{2,1}]_2 \]

  \[ r_{17} \equiv [\tilde{d}_2]_2 \rightarrow [\tilde{e}_2 f_{2,2}]_2 \]

  - Preferences

  \[ r_{18} \equiv [\tilde{d}_3]_2 \rightarrow [\tilde{e}_3 f_{3,1}]_2 \]

  \[ r_{19} \equiv [\tilde{d}_3]_2 \rightarrow [\tilde{e}_3 f_{3,2}]_2 \]

  Solution: \( f_{i,j} \) follows the probability distribution of the desired transactions probabilities.

- **Step 4: Purchase transactions**

  \[ r_{20} \equiv [d_{i} \tilde{e}_j f_{i,j} v_{j}^{k_{i,j}}]_2 \rightarrow [b_{i} c_{j} u_{i}^{k_{i,j}}]_2 \quad 1 \leq i \leq k_1, 1 \leq j \leq k_2 \]

  - \( k_1 \): total number of producers.

  - \( k_2 \): total number of retailers.

  - \( k_{i,j} \): price fixed by producer \( i \) for each unit of \( d_i \).
Set of rules – Cleaning

Step 5: cleaning rules

- Eliminate non-exhausted authorizations:
  \[ r_{26} \equiv [f_{i,j}]_2^{-} \rightarrow [ ]_2^0 \quad 1 \leq i \leq k_1, 1 \leq j \leq k_2 \]

- Unauthorize non-exhausted \( \tilde{e}_j \):
  \[ r_{27} \equiv [\tilde{e}_j]_2^{-} \rightarrow \tilde{d}_j[ ]_2^0 \quad 1 \leq j \leq k_2 \]

- Signaling a new cycle:
  \[ r_{30} \equiv [r_1, r_2]_2^{-} \rightarrow r_1, r_2 [ ]_2^0 \]

\( k_1 \): total number of producers.

\( k_2 \): total number of retailers.
P - Lingua

- Set of rules has been implemented in P – Lingua.

- An example for each set of rules:
  - Initialization:
    
    ```
    /* r2 */ s, R_1[ ]'2 → s, a * k{3} + [R_1]'2 :: k_{2,4};
    ```
  - Production:
    
    ```
    /* r9 */ b{i}, a, u{i} * k{7} + [ ]'2 → u s * k{7}[ d{i}]'2 :: 1 : 1 ≤ i ≤ k{1}
    ```
  - Authorization:
    
    ```
    /* r18 */ [d{n}{3} ]'2 → [e{n}{3}, f{3,1} ]'2 :: 0.15
    ```
  - Transaction:
    
    ```
    /* r20 */ [d{i}, e{n}{j}, f{j, i}, v{j} * k{i, 6} ]'2 → −[ b{i}, c{j}, u{i} * k{i, 6} ]'2 :: 1 1 ≤ i ≤ k{1}, 1 ≤ j ≤ k{2}
    ```
Simplified trace

STEP 1:
- Generic demand generation
- Supply creation

STEP 2:
- Production of goods
- Order generation

STEP 3:
- Generation of purchase transaction authorizations

STEP 4:
- Purchase transactions

STEP 5:
- Cleaning
- Technical rules

Membrane Computing Applications in Computational Economics
Simulation parameters

- Simulation tool: MeCoSim
- Parameters: same as Păun’s paper

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value/s</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_1$</td>
<td>2</td>
<td>Total number of producers</td>
</tr>
<tr>
<td>$k_2$</td>
<td>3</td>
<td>Total number of retailers</td>
</tr>
<tr>
<td>$k_3$</td>
<td>60</td>
<td>Units of raw material inserted into the system by $S$</td>
</tr>
<tr>
<td>$k_4$</td>
<td>1</td>
<td>Deviation from $k_3$</td>
</tr>
<tr>
<td>$k_5$</td>
<td>60</td>
<td>Units of aggregate demand inserted into the system by $C$</td>
</tr>
<tr>
<td>$k_6$</td>
<td>1</td>
<td>Deviation from $k_5$</td>
</tr>
<tr>
<td>$k_7$</td>
<td>11</td>
<td>Price fixed by $S$ for each unit of a</td>
</tr>
<tr>
<td>$k_8$</td>
<td>14</td>
<td>Price fixed by $C$ as an estimation of each order of good</td>
</tr>
<tr>
<td>$k_{i,1}$</td>
<td>(65,35)</td>
<td>Initial production capacity of producer $i$. $1 \leq i \leq k_1$</td>
</tr>
<tr>
<td>$k_{i,2}$</td>
<td>(750,400)</td>
<td>Initial monetary units of producer $i$. $1 \leq i \leq k_1$</td>
</tr>
<tr>
<td>$k_{j,3}$</td>
<td>(50,30,20)</td>
<td>Initial capacity of retailer $j$. $1 \leq j \leq k_2$</td>
</tr>
<tr>
<td>$k_{m,4}$</td>
<td>(0.01,0.95,0.03)</td>
<td>Values of discrete probability distribution of units of raw material inserted into the system by $S$</td>
</tr>
<tr>
<td>$k_{m,5}$</td>
<td>(0.03,0.90,0.04)</td>
<td>Values of discrete probability distribution of units of aggregate demand inserted into the system by $C$</td>
</tr>
<tr>
<td>$k_{i,6}$</td>
<td>(12,13)</td>
<td>Price fixed by producer $i$ for each unit of $d_i$</td>
</tr>
<tr>
<td>$k_{j,7}$</td>
<td>(13,14,15)</td>
<td>Price fixed by retailer $j$ for each order of good $j$. $1 \leq j \leq k_2$</td>
</tr>
</tbody>
</table>
## MeCoSim definition

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_1$</td>
<td>$[@r,1&gt;$</td>
<td>Captures number of producers based on the number of rows in table Producer_input</td>
</tr>
<tr>
<td>$k_2$</td>
<td>$[@r,8&gt;$</td>
<td>Captures number of retailers based on the number of rows in table Retailer_input</td>
</tr>
<tr>
<td>$k_3$</td>
<td>$&lt;$9,$1$-2,2$&gt;</td>
<td>Units of raw material inserted into the system by $S$</td>
</tr>
<tr>
<td>$k_4$</td>
<td>Deviation from $k_3$</td>
<td></td>
</tr>
<tr>
<td>$k_5$</td>
<td>$&lt;$9,$1$-$2,2$&gt;</td>
<td>Units of aggregate demand inserted into the system by $C$</td>
</tr>
<tr>
<td>$k_6$</td>
<td>Index $1 = [3..&lt;@r,9&gt;+2]$</td>
<td>Deviation from $k_5$</td>
</tr>
<tr>
<td>$k_7$</td>
<td>Price fixed by $S$ for each unit of a</td>
<td></td>
</tr>
<tr>
<td>$k_8$</td>
<td>Price fixed by $C$ as an estimation of each order of good</td>
<td></td>
</tr>
<tr>
<td>$k_{i,1}$</td>
<td>$&lt;$1,$15$,$1$+$3$&gt;</td>
<td>Initial production capacity of producer $i$. $1 \leq i \leq k_1$</td>
</tr>
<tr>
<td>$k_{i,2}$</td>
<td>Index $1 = [1..k[1]]$</td>
<td>Initial monetary units of producer $i$. $1 \leq i \leq k_1$</td>
</tr>
<tr>
<td>$k_{j,3}$</td>
<td>$&lt;$8,$1$,5$&gt;</td>
<td>Initial capacity of retailer $j$. $1 \leq j \leq k_2$</td>
</tr>
<tr>
<td>$k_{m,4}$</td>
<td>$&lt;$10,$1$,5$,$2$-$3$&gt;</td>
<td>Values of discrete probability distribution of units of raw material inserted into the system by $S$</td>
</tr>
<tr>
<td>$k_{m,5}$</td>
<td>Index $1 = [1..&lt;@r,10&gt;]$</td>
<td>Values of discrete probability distribution of units of aggregate demand inserted into the system by $C$</td>
</tr>
<tr>
<td>$k_{i,6}$</td>
<td>$&lt;$1,$15$,$6$&gt;</td>
<td>Price fixed by producer $i$ for each unit of $d_i$</td>
</tr>
<tr>
<td>$k_{j,7}$</td>
<td>$&lt;$8,$15$,$5$&gt;</td>
<td>Price fixed by retailer $j$ for each order of good $j$. $1 \leq j \leq k_2$</td>
</tr>
</tbody>
</table>
Simulation results – monetary units

Producers’ monetary units

Retailers’ monetary units
Simulation results - capacities

Producers’ capacities

Retailers’ capacities
Simulation results - comparison

Păun's evolution

Initial model evolution
Enhanced Model

Summarized behavior of Initial Model:
- A steady increase of monetary units owned by producers, retailers and generic consumer.
- Nearly stable producer’s and retailer’s capacities.
- Monetary units obtained by raw source of material get out of circulation.

Why?
- Producers’ & retailers’ capacities are fixed and no changes are allowed.
- Raw material and aggregate demand are initially settled and remain unchanged during the system evolution.
- Artificial exogenous injection of monetary units into consumer $C$ at the beginning of each cycle. This flow is necessary to maintain system evolving.
Getting closer to real situations:

- Allowing variations of producers’ and retailers’ capacities:
  - Capital stock depreciation.
  - Investment or capital increase decision.

- Remove external injection of monetary units:
  - Payment of rents to the owners of the production factors.
  - Raw material source is owned by the aggregate consumer.
  - Aggregate consumer is stakeholder of producers and retailers, thus implying dividends payments.

- Inclusion of randomness in a PDP-way:
  - Raw material generation.
  - Aggregate demand generation.
  - Mechanism of capacity increase decision.
Producer – Retailer Enhanced Model

Membrane Computing Applications in Computational Economics
Model Entities

- **Actors:**
  - Producers: \((b_i, u_i) \rightarrow \text{(capacity, money)}\)
  - Retailers: \((c_j, v_j) \rightarrow \text{(capacity, money)}\)

- **Generic sources:**
  - Of raw material \((u_S, \text{generation rate})\)
  - Of demand or Generic consumer \((u_C, \text{demand rate})\)
Model interactions

- **Good’s and order’s flows:**
  - Producers generate good \( d \) from raw material.
  - Retailers receive order \( \bar{d} \) from generic consumer.
  - \( d \) and \( \bar{d} \) are matched
  - Purchase \( P_{i,j} = P(producer \ i, \ retailer \ j) \)

- **External monetary injection:**
  - Removed.
Additional interactions

- **Monetary flows:**
  - Initial Model monetary exchange due to prices.
  - Rents payments to owners: Generic Consumer.
  - Dividends payments to stakeholders: Generic Consumer.
  - Raw material source owners: Generic Consumer.

- **Capacity variations:**
  - Producers’ capacity depreciation.
  - Producers’ capacity increase decision: non-satisfied demand from retailers.
Defining steps of cycle

INITIALIZATION
- Capacity cost payments
- Aggregate demand creation
- Raw material disposability

EVOLUTION
- Dividend payments
- Capacity increase decision
- Capacity depreciation

PRODUCTION
- Orders generation
- Production of goods

TRANSACTION
- Purchase transactions

AUTHORIZATION
- Purchase authorizations generation
Model Formalization (I)

\[ \Pi = (G, \Gamma, \Sigma, T, \mathcal{R}_E, \mu, \mathcal{R}_\Pi, \{ f_\Gamma \in \mathcal{R}_\Pi \}, M_1, M_2) \quad \text{PDP System of degree (2,1)} \]

Where:

- \( G = (V, E) \) with \( V = \{e_1\} \) and \( E = \{(e_1, e_1)\} \).
- Working alphabet: \( \Gamma_{\text{enhanced}} = \Gamma_{\text{initial}} \cup \{g_i, y_i, m_i, z_i, h_i: 1 \leq i \leq k_1\} \cup \{p, q\} \)

Where:

- \( C \): aggregate generic consumer.
- \( S \): raw material supplier.
- \( \bar{d} \): unit of aggregate demand from \( C \).
- \( a \): unit of supplied raw material provided by \( S \).
- \( u_C \): monetary unit owned by \( C \).
- \( b_i \): unit of production capacity of producer \( i \). \( 1 \leq i \leq k_1 \).
- \( d_i \): unit of good supplied by producer \( i \). \( 1 \leq i \leq k_1 \).
- \( u_i \): monetary unit owned by producer \( i \). \( 1 \leq i \leq k_1 \).
- \( c_j \): unit of capacity of retailer \( j \). \( 1 \leq j \leq k_2 \).
- \( \bar{d}_j \): unit of good demanded by retailer \( j \). \( 1 \leq j \leq k_2 \).
- \( v_j \): monetary unit owned by retailer \( j \). \( 1 \leq j \leq k_2 \).
- \( \bar{e}_j \): unit of good demanded by retailer and authorized for transaction unit of \( \bar{d}_j \). \( 1 \leq j \leq k_2 \).
- \( f_{i,j} \): authorization for \( \bar{d}_j \) to be exchange with \( d_i \). \( 1 \leq i \leq k_1 \), \( 1 \leq j \leq k_2 \).
- \( R_1 \): for technical reasons.
- \( p \): randomness generator for \( a \) provision by \( S \).
- \( q \): randomness generator for \( \bar{d} \) generation by \( C \).
- \( h_i \): unit of production capacity of producer \( i \) before depreciation. \( 1 \leq i \leq k_1 \).
- \( y_i \): unit (in idle state) of aborted purchase transactions considered for capacity increase. \( 1 \leq i \leq k_1 \).
- \( m_i \): randomness generator for \( y_i \). \( 1 \leq i \leq k_1 \).
- \( z_i \): activated unit of aborted purchase transactions considered for capacity increase. \( 1 \leq i \leq k_1 \) \( . 1 \leq i \leq k_1 \).
- \( g_i \): for technical reasons. \( 1 \leq i \leq k_1 \).
Model Formalization (II)

- $\Sigma = \emptyset$.
- $R_E = \emptyset$.
- $\Pi = \{\Gamma, \mu, M_1, M_2, R_\Pi\}$ where:
  - Membrane structure: $\mu = [[ ]]_1$.
  - $M_1 = \{C, S, R_1\} \cup \{g_i, u_i^{k_{i,1}k_{10}*7}: 1 \leq i \leq k_1\}, \{v_j^{k_{j,3}k_{10}*7}: 1 \leq j \leq k_2\}$
  - $M_2 = \{c_j^{k_{j,3}}: 1 \leq j \leq k_2\} \cup \{b_i^{k_{i,1}}: 1 \leq i \leq k_1\}$

Initial multisets contain basically:

- $b_i^{k_{i,1}}, u_i^{k_{i,1}k_{10}*7}$: producers’ initial parameters.
- $c_j^{k_{j,3}}, v_j^{k_{j,3}k_{10}*7}$: retailers’ initial parameters.

They need same initial amount of monetary units to pay initial capacity costs. Where:

- $k_{i,1}$: initial production capacity of producer $i$. $1 \leq i \leq k_1$.
- $k_{j,3}$: initial capacity of retailer $j$. $1 \leq j \leq k_2$. 
Model Parameters

- **Goal:** maximize model parametrization

  - $k_1$: total number of producers.
  - $k_2$: total number of retailers.
  - $k_3$: raw material inserted into the system by $S$ – minimum value of range
  - $k_4$: raw material inserted into the system by $S$ – maximum value of range.
  - $k_5$: aggregate demand inserted into the system by $C$ – minimum value of range.
  - $k_6$: aggregate demand inserted into the system by $C$ – maximum value of range.
  - $k_7$: price fixed by $S$ for each unit of $a$.
  - $k_8$: number of failed purchases considered for the analysis of increasing capital stock – minimum value.
  - $k_9$: number of failed purchases considered for the analysis of increasing capital stock – maximum value.
  - $k_{10}$: cost of capital stock per cycle.
  - $k_{11}$: depreciation rate of capital stock.
  - $k_{12}$: step of capacity increase.
  - $k_{13}$: dividend percentage.
  - $k_{i,1}$: initial production capacity of producer $i$. $1 \leq i \leq k_1$.
  - $k_{i,2}$: price fixed by producer $i$ for each unit of $d_i$. $1 \leq i \leq k_1$.
  - $k_{j,3}$: initial capacity of retailer $j$. $1 \leq j \leq k_2$.
  - $k_{i,6}$: price fixed by retailers $j$ for each order of good. $1 \leq j \leq k_2$. 
Set of rules – Initialization

- From Naïve randomness:

  \[ r_5 \equiv R_2 \ c[ ]_2^{k_{1,5}} \overset{\bar{d}^{k_5+k_6}}{\longrightarrow} c[ R_2]_2^+ \]
  \[ r_6 \equiv R_2 \ c[ ]_2^{k_{2,5}} \overset{\bar{d}^{k_5}}{\longrightarrow} c[ R_2]_2^+ \]
  \[ r_7 \equiv R_2 \ c[ ]_2^{k_{3,5}} \overset{\bar{d}^{k_5-k_6}}{\longrightarrow} c[ R_2]_2^+ \]
  \[ r_8 \equiv R_2 \ c[ ]_2^{1-k_{1,5}-k_{2,5}-k_{3,5}} \overset{\bar{d}^{k_5-2*k_6}}{\longrightarrow} c[ R_2]_2^+ \]

  \[ \text{Generates } \bar{d} \text{ around } k_5 \]

- To a PDP-way: raw material disposability & generic demand creation:

  \[ r_1 \equiv R_1 \ s \ c[ ]_2 \rightarrow a^{k_3} \ p^{k_4-k_3} \ \bar{d}^{k_5} \ q^{k_6-k_5} \ s \ c[ R_1]_2^+ \]
  \[ r_2 \equiv p \ [ ]_2^{-0.5} \rightarrow [ ]_2^+ \]
  \[ r_3 \equiv p \ [ ]_2^{-0.5} \rightarrow a \ [ ]_2^+ \]
  \[ r_4 \equiv q \ [ ]_2^{-0.5} \rightarrow [ ]_2^+ \]
  \[ r_5 \equiv q \ [ ]_2^{-0.5} \rightarrow \bar{d} \ [ ]_2^+ \]

  \[ \text{Generates } [\bar{d}^{k_5}, \bar{d}^{k_6}] \]
  \[ \text{Generates } [a^{k_3}, a^{k_4}] \]
Set of rules – Capacity costs

- Rents for capacity:
  - Generic consumer is the owner of production factors.
  - Agents have enough monetary units to pay for capacity:
    \[
    r_9 \equiv u_i^{k_{10}} [b_i]_2 \rightarrow b_i \ u_c^{k_{10}} [ ]_2^+ 1 \leq i \leq k_1
    \]
    \[
    r_{10} \equiv v_j^{k_{10}} [c_j]_2 \rightarrow c_j \ u_c^{k_{10}} [ ]_2^+ 1 \leq j \leq k_2
    \]
  - Agents are not able to pay for capacity:
    \[
    r_{11} \equiv [b_i]^+ \rightarrow u_c^{k_{10}} [ ]_2 1 \leq i \leq k_1
    \]
    \[
    r_{12} \equiv [c_j]^+ \rightarrow u_c^{k_{10}} [ ]_2 1 \leq j \leq k_2
    \]

\[k_1: \text{total number of producers.}\]
\[k_2: \text{total number of retailers.}\]
\[k_{10}: \text{cost of capital stock per cycle.}\]
Set of rules – Operations

- Main changes:
  - Generic consumer is the owner of raw material source

- Producer operation:
  \[ r_{14} \equiv a \ b_i \ u_i^{k_7} \ [ ]_2^+ \rightarrow u_c^{k_7} \ [ d_i ]_2^0 \ 1 \leq i \leq k_1 \]

- Retailer operation:
  \[ r_{15} \equiv d \ c_j \ u_c^{k_j,6} \ [ ]_2^+ \rightarrow [ d_j v_j^{k_j,6} ]_2^0 \ 1 \leq j \leq k_2 \]

- Unused capacities:
  \[ r_{16} \equiv b_i \ [ ]_2 \rightarrow [ b_i ]_2 \ 1 \leq i \leq k_1 \]
  \[ r_{17} \equiv c_j \ [ ]_2 \rightarrow [ c_j ]_2 \ 1 \leq j \leq k_2 \]

\[ k_1: \text{total number of producers.} \]
\[ k_2: \text{total number of retailers.} \]
\[ k_7: \text{price fixed by } S \text{ for each unit of } a. \]
\[ k_{i,6}: \text{price fixed by retailers } j \text{ for each order of good.} \]
\[ k_{j,7}: \text{price fixed by retailers } j \text{ for each order of good.} \]

Retired from the operational membrane waiting for their depreciation.
Set of rules – Auth. & Transactions

Purchase authorization generation

- **Geo-barriers**
  
  \[
  r_{18} \equiv [\tilde{d}_1^1]_2 \rightarrow [\tilde{e}_1 f_{1,1}^1]_2 \\
  r_{19} \equiv [\tilde{d}_1^1]_2 \rightarrow [\tilde{e}_1 f_{1,2}^1]_2 \\
  r_{20} \equiv [\tilde{d}_2^2]_2 \rightarrow [\tilde{e}_2 f_{2,1}^2]_2 \\
  r_{21} \equiv [\tilde{d}_2^2]_2 \rightarrow [\tilde{e}_2 f_{2,2}^2]_2 \\
  r_{22} \equiv [\tilde{d}_3^3]_2 \rightarrow [\tilde{e}_3 f_{3,1}^3]_2 \\
  r_{23} \equiv [\tilde{d}_3^3]_2 \rightarrow [\tilde{e}_3 f_{3,2}^3]_2
  \]

- **Non-preferences**

- **Preferences**

Purchase transactions

\[
  r_{24} \equiv [d_i \tilde{e}_j f_{j,i} v_{j,i}^{k_{i,2}}]_2^0 \rightarrow u_i^{k_{i,2}} [h_i c_j]_2^1 \quad 1 \leq i \leq k_1, \ 1 \leq j \leq k_2
\]

\(b_i\) are retired as \(h_i\) from the operational membrane waiting for their depreciation.

\(k_1\): total number of producers.

\(k_2\): total number of retailers.

\(k_{i,2}\): price fixed by producer \(i\) for each unit of \(d_i\).
Set of rules - Evolution

- **Dividend payment:**

  \[ r_{25} \equiv \left[ v_j \right]_2 \rightarrow v_j \left[ \right]_2^0 \quad 1 \leq j \leq k_2 \]

  \[ r_{26} \equiv u_i \left[ \right]_2 \xrightarrow{k_{13}} u_C \left[ \right]_2^0 \quad 1 \leq i \leq k_1 \]

  \[ r_{27} \equiv u_i \left[ \right]_2 \xrightarrow{1-k_{13}} u_i \left[ \right]_2^0 \quad 1 \leq i \leq k_1 \]

  Both blocks of rules only applied to producers

- **Capacity depreciation:**

  \[ r_{31} \equiv \left[ h_i \right]_2 \xrightarrow{1-k_{11}} \left[ b_i \right]_2^0 \quad 1 \leq i \leq k_1 \]

  \[ r_{32} \equiv \left[ h_i \right]_2 \xrightarrow{k_{11}} \left[ \right]_2^0 \quad 1 \leq i \leq k_1 \]

  \[ k_1: \text{total number of producers.} \]

  \[ k_2: \text{total number of retailers.} \]

  \[ k_{11}: \text{depreciation rate of capital stock.} \]

  \[ k_{13}: \text{dividend percentage.} \]
Set of rules – capacity increase

- When strictly necessary only

- Trigger: non-exhausted $f_{j,i}$
  
  - Case a: Enough producer capacity:
    
    $$r_{28} \equiv [f_{j,i} \, d_i]^2_2 \rightarrow [d_i]^0_2 \quad 1 \leq i \leq k_1, 1 \leq j \leq k_2$$
    $$r_{29} \equiv [f_{j,i} \, h_i]^2_2 \xrightarrow{1-k_{11}} [b_i]^0_2 \quad 1 \leq i \leq k_1, 1 \leq j \leq k_2$$
    $$r_{30} \equiv [f_{j,i} \, h_i]^2_2 \xrightarrow{k_{11}} [\_]^0_2 \quad 1 \leq i \leq k_1, 1 \leq j \leq k_2$$

  - Case b: Not enough producer capacity:
    
    $$r_6 \equiv g_i \, [\_]^0_2 \rightarrow [g_1 \, y_i \, k_8 \, m_i^{(k_9-k_8)}]^+_2 \quad 1 \leq i \leq k_1$$
    $$r_7 \equiv [m_i]^+_2 \rightarrow [\_]^0_2 \quad 1 \leq i \leq k_1$$
    $$r_8 \equiv [m_i]^+_2 \xrightarrow{0.5} [y_i]^0_2 \quad 1 \leq i \leq k_1$$
    $$r_{33} \equiv [y_i]^+_2 \rightarrow [z_i]^0_2 \quad 1 \leq i \leq k_1$$
    $$r_{34} \equiv [f_{j,i} \, z_i]^0_2 \rightarrow b_i^{-k_{12}}[\_]^+_2 \quad 1 \leq i \leq k_1, 1 \leq j \leq k_2$$

$k_1$: total number of producers.

$k_2$: total number of retailers.

$k_8$: number of failed purchases considered for the analysis of increasing capital stock – min value.

$k_9$: number of failed purchases considered for the analysis of increasing capital stock – max value.

$k_{11}$: depreciation rate of capital stock.

Generates $[y_i^{k_8}, y_i^{k_9}]$
Set of rules – Cleaning

Cleaning rules and technical rules

- Eliminate non-exhausted authorizations:

  \[ r_{35} \equiv [f_{j,i}]_{2}^{+} \rightarrow [ ]_{2}^{0} \quad 1 \leq i \leq k_{1}, 1 \leq j \leq k_{2} \]

  \[ r_{36} \equiv [z_{i}]_{2}^{+} \rightarrow [ ]_{2}^{0} \quad 1 \leq i \leq k_{1} \]

  \[ k_{1} \text{: total number of producers.} \]

  \[ k_{2} \text{: total number of retailers.} \]

- Unauthorized non-exhausted \( \bar{e}_{j} \):

  \[ r_{13} \equiv v_{j} [ ]_{2}^{+} \rightarrow [v_{j}]_{2}^{0} \quad 1 \leq j \leq k_{2} \]

  \[ r_{37} \equiv [\bar{e}_{j}]_{2}^{+} \rightarrow [\bar{d}_{j}]_{2}^{0} \quad 1 \leq j \leq k_{2} \]

- Signaling a new cycle:

  \[ r_{38} \equiv [r_{1}]_{2}^{-} \rightarrow r_{1} [ ]_{2}^{0} \]

  \[ r_{39} \equiv [g_{i}]_{2}^{-} \rightarrow g_{i} [ ]_{2}^{0} \quad 1 \leq j \leq k_{2} \]
P - Lingua

- Set of rules has been implemented in P – Lingua.
- An example for each set of rules:
  - Initialization:
    ```
    /* r1 */   s, c, r1 [ ]’2 → s, c, a * k{3}, p * (k{4} − k{3}), dn * k{5}, q * (k{6} − k{5}) +[r1]’2:: 1
    ```
  - Production:
    ```
    /* r9 */   u{i} * k{10} [b{i}]’2 → b{i}, uc * k{10} + [ ]’2 :: 1 : 1 <= i <= k{1};
    ```
  - Transaction:
    ```
    /* r24 */   [d{i}, en{j}, f{j}, i], v{j} * k{i, 2} ]’2 → u{i} * k{i, 2} − [ h{i}, c{j}, ]’2 :: 1 : 1 <= i <= k{1}, 1 ≤ j ≤ k{2}
    ```
  - Capacity increase:
    ```
    /* r34 */   [ f{j, i}, z{i}]’2 → b{i} * k{12} + [ ]’2 :: 1 : 1 ≤ i ≤ k{1}, 1 ≤ j ≤ k{2}
    ```
Simplified trace

STEP 1:
• Generic demand generation
• Supply creation
• Capacity cost payment

STEP 2:
• Production of goods
• Order generation

STEP 3:
• Generation of purchase transaction authorizations

STEP 4:
• Purchase transactions

STEP 5:
• Dividend payment
• Capacity depreciation
• Capacity increase decision
### Simulation parameters

Parameters: similar to Păun’s paper

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_1$</td>
<td>2</td>
<td>Total number of producers</td>
</tr>
<tr>
<td>$k_2$</td>
<td>3</td>
<td>Total number of retailers</td>
</tr>
<tr>
<td>$k_3$</td>
<td>59</td>
<td>Units of raw material inserted into the system by $S$ — minimum value of range</td>
</tr>
<tr>
<td>$k_4$</td>
<td>62</td>
<td>Units of raw material inserted into the system by $S$ — maximum value of range</td>
</tr>
<tr>
<td>$k_5$</td>
<td>59</td>
<td>Units of aggregate demand inserted into the system by $C$ — minimum value of range</td>
</tr>
<tr>
<td>$k_6$</td>
<td>62</td>
<td>Units of aggregate demand inserted into the system by $C$ — maximum value of range</td>
</tr>
<tr>
<td>$k_7$</td>
<td>11</td>
<td>Price fixed by $S$ for each unit of a</td>
</tr>
<tr>
<td>$k_8$</td>
<td>3</td>
<td># failed purchases considered for the analysis of increasing capital stock — minimum value.</td>
</tr>
<tr>
<td>$k_9$</td>
<td>5</td>
<td># failed purchases considered for the analysis of increasing capital stock — maximum value.</td>
</tr>
<tr>
<td>$k_{10}$</td>
<td>2</td>
<td>cost of capital stock per cycle</td>
</tr>
<tr>
<td>$k_{11}$</td>
<td>0.1</td>
<td>depreciation rate of capital stock</td>
</tr>
<tr>
<td>$k_{12}$</td>
<td>1</td>
<td>step of capacity increase</td>
</tr>
<tr>
<td>$k_{13}$</td>
<td>0.01</td>
<td>Dividend percentage</td>
</tr>
<tr>
<td>$k_{i,1}$</td>
<td>(65,35)</td>
<td>Initial production capacity of producer $i$. $1 \leq i \leq k_1$</td>
</tr>
<tr>
<td>$k_{i,2}$</td>
<td>(13,13)</td>
<td>Price fixed by producer $i$ for each unit of $d_i$. $1 \leq i \leq k_1$</td>
</tr>
<tr>
<td>$k_{j,3}$</td>
<td>(50,30,20)</td>
<td>Initial capacity of retailer $j$. $1 \leq j \leq k_2$</td>
</tr>
<tr>
<td>$k_{i,6}$</td>
<td>(15,15,15)</td>
<td>Price fixed by retailer $j$ for each order of good $j$. $1 \leq j \leq k_2$</td>
</tr>
</tbody>
</table>
## MeCoSim definition

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_1$</td>
<td>&lt;@r,1&gt; (\text{Index 1 = 1})</td>
<td>Captures number of producers based on the number of rows in table Producer_input</td>
</tr>
<tr>
<td>$k_2$</td>
<td>&lt;@r,8&gt; (\text{Index 2 = 2})</td>
<td>Captures number of retailers based on the number of rows in table Retailer_input</td>
</tr>
<tr>
<td>$k_3$</td>
<td>(k_3) (\text{Index 1} = 1, \text{Index 2} = 2)</td>
<td>Units of raw material inserted into the system by (S) – minimum value of range</td>
</tr>
<tr>
<td>$k_4$</td>
<td>(k_4) (\text{Index 1} = 1, \text{Index 2} = 2)</td>
<td>Units of raw material inserted into the system by (S) – maximum value of range</td>
</tr>
<tr>
<td>$k_5$</td>
<td>(k_5) (\text{Index 1} = 1, \text{Index 2} = 2)</td>
<td>Units of aggregate demand inserted into the system by (C) – minimum value of range</td>
</tr>
<tr>
<td>$k_6$</td>
<td>(k_6) (\text{Index 1} = 1, \text{Index 2} = 2)</td>
<td>Units of aggregate demand inserted into the system by (C) – maximum value of range</td>
</tr>
<tr>
<td>$k_7$</td>
<td>(k_7) (\text{Index 1} = [3..&lt;@r,9&gt;+2])</td>
<td>Price fixed by (S) for each unit of a</td>
</tr>
<tr>
<td>$k_8$</td>
<td>(k_8) (\text{Index 1} = [3..&lt;@r,9&gt;+2])</td>
<td># failed purchases considered for the analysis of increasing capital stock – minimum value.</td>
</tr>
<tr>
<td>$k_9$</td>
<td>(k_9) (\text{Index 1} = [3..&lt;@r,9&gt;+2])</td>
<td># failed purchases considered for the analysis of increasing capital stock – maximum value.</td>
</tr>
<tr>
<td>$k_{10}$</td>
<td>(k_{10}) (\text{Index 1} = 1, \text{Index 2} = 2)</td>
<td>Cost of capital stock per cycle</td>
</tr>
<tr>
<td>$k_{11}$</td>
<td>(k_{11}) (\text{Index 1} = 1, \text{Index 2} = 2)</td>
<td>Depreciation rate of capital stock</td>
</tr>
<tr>
<td>$k_{12}$</td>
<td>(k_{12}) (\text{Index 1} = 1, \text{Index 2} = 2)</td>
<td>Step of capacity increase</td>
</tr>
<tr>
<td>$k_{13}$</td>
<td>(k_{13}) (\text{Index 1} = 1, \text{Index 2} = 2)</td>
<td>Dividend percentage</td>
</tr>
<tr>
<td>$k_{i1}$</td>
<td>(k_{i1}) (\text{Index 1} = [1..k_1])</td>
<td>Initial production capacity of producer (i). (1 \leq i \leq k_1)</td>
</tr>
<tr>
<td>$k_{i2}$</td>
<td>(k_{i2}) (\text{Index 1} = [1..k_1])</td>
<td>Price fixed by producer (i) for each unit of (d_i). (1 \leq i \leq k_1)</td>
</tr>
<tr>
<td>$k_{j3}$</td>
<td>(k_{j3}) (\text{Index 1} = [1..k_2])</td>
<td>Initial capacity of retailer (j). (1 \leq j \leq k_2)</td>
</tr>
<tr>
<td>$k_{i6}$</td>
<td>(k_{i6}) (\text{Index 1} = [1..k_2])</td>
<td>Price fixed by retailer (j) for each order of good (j). (1 \leq j \leq k_2)</td>
</tr>
</tbody>
</table>
Simulation results – capacities

Producers’ capacities with:

- Depreciation rate = 0.1
- Deactivated capacity increase mechanism.

Producers’ capacities with:

- Depreciation rate = 0.1
- Activated capacity increase mechanism.
Simulation results – dividends

Generic consumer monetary units:
• Deactivated dividend payment.

Generic consumer monetary units:
• Restored dividend payment.
Simulation results – producer

- Initial distribution of capacities: 65 + 35
- Raw material generation rate [59,62]
- Raw material generation rate [40,43]

System tries to reach an equilibrium point in function of parameters of $S$
Simulation results – retailer

- Initial distribution of capacities: 50 + 30 + 20
- Generic demand generation rate [59,62]
- Generic demand generation rate [40,43]

System tries to reach an equilibrium point in function of parameters of $C$
CONCLUSIONS

Initial model:
- We have been able to reproduce Păun’s results using:
  - PDP systems.
  - P – Lingua & MeCoSim framework.
  - Inference engine DNDP4.

Enhanced model:
- Initial model has been extended including several real world economic processes.
  - Cost of production factors, dividend payment.
  - Capacity depreciation, capacity increase mechanisms.
  - Removing external injection of monetary units.
- Model evolves autonomously around an equilibrium point different from the initial conditions.
FURTHER DEVELOPMENTS

- Complete the enhanced model:
  - Macroeconomics interest: behavior of system under perturbation around equilibrium.
  - Introduce mechanisms to adjust prices to some stimulus.
  - Investigate if different patterns of randomness could be generated easily.

- Future Case of Study:
  - SDGE (Stochastic Dynamic General Equilibrium).
  - Previous techniques can be utilized in this problem.
  - Challenge: generate an emergent optimization behavior.

