Introductory Overview on Polymorphic P Systems

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19th BWMC, Sevilla, January 26, 2023

The papers

- Artiom Alhazov, Sergiu Ivanov, Yurii Rogozhin: Polymorphic P Systems.
  In: *CMC 2010*, Vol. 6501 of *LNCS*, pp. 81-94, 2010
- Sergiu Ivanov:
  Polymorphic P Systems with Non-cooperative Rules and No Ingredients.
  In: *CMC 2014*, Vol. 8961 of *LNCS*, pp. 258-273, 2014

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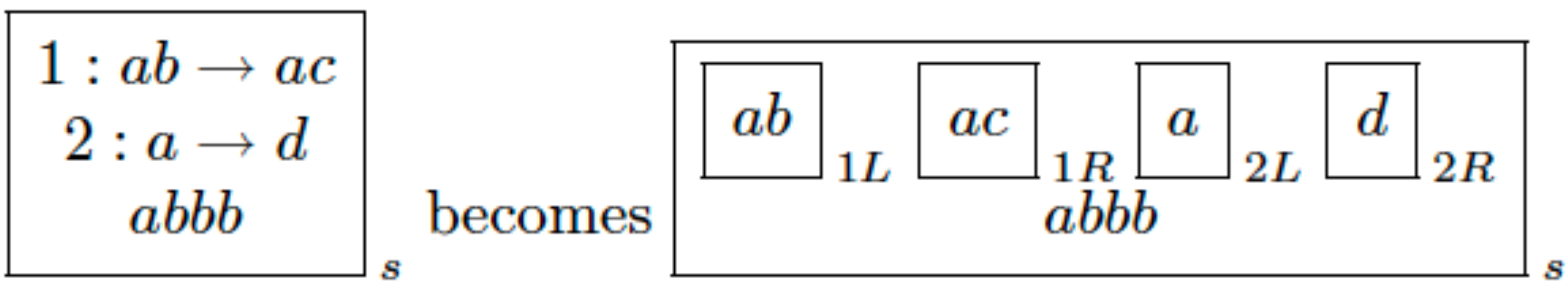
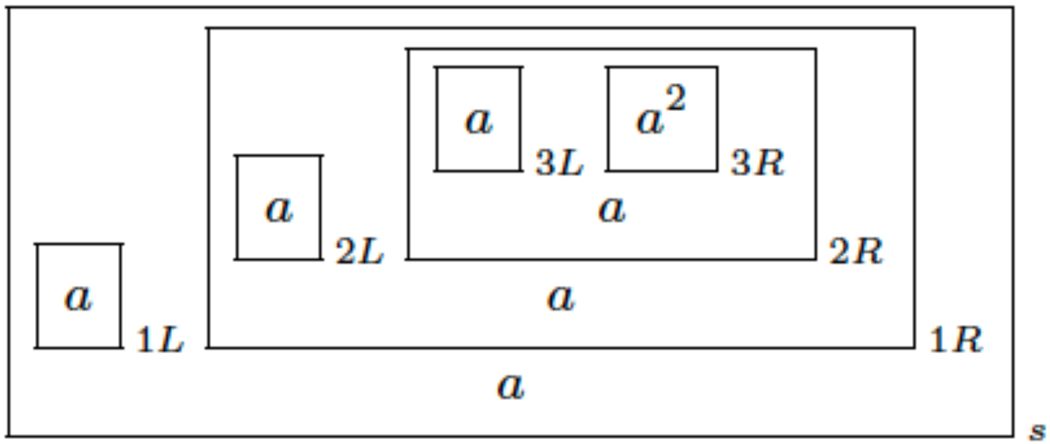
The idea

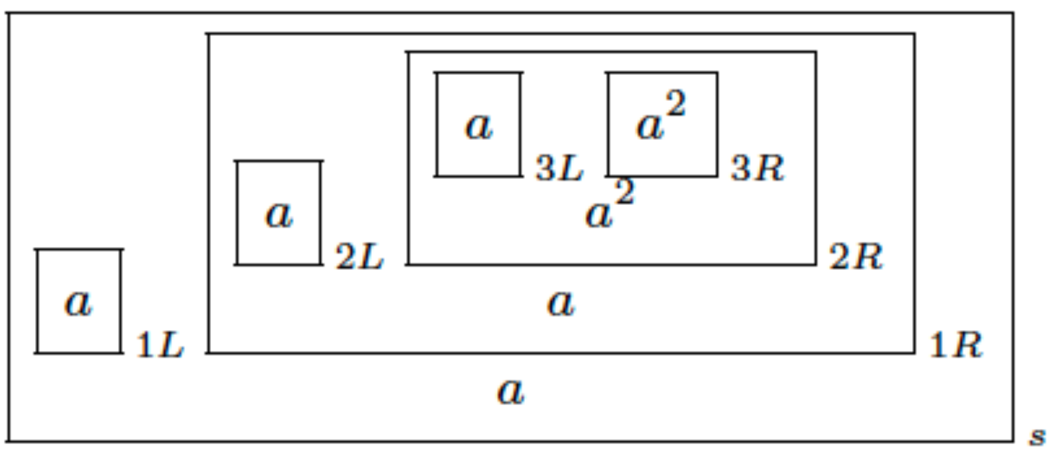
- To manipulate the rules during a computation: represent them as data



For example



$3 : a \rightarrow a^2$ in $2R$
$2 : a \rightarrow a$ in $1R$
$1 : a \rightarrow a$ in s
$\Rightarrow$



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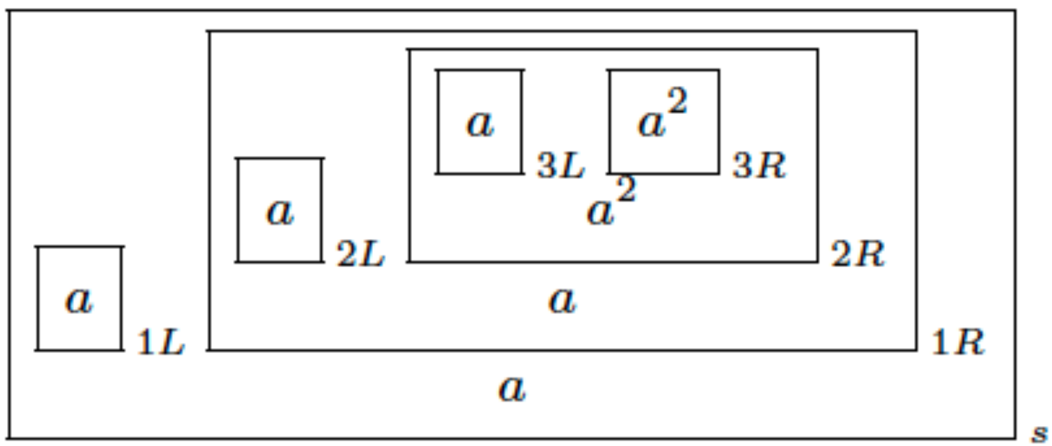

$3 : a \to a^2$ in $2R$
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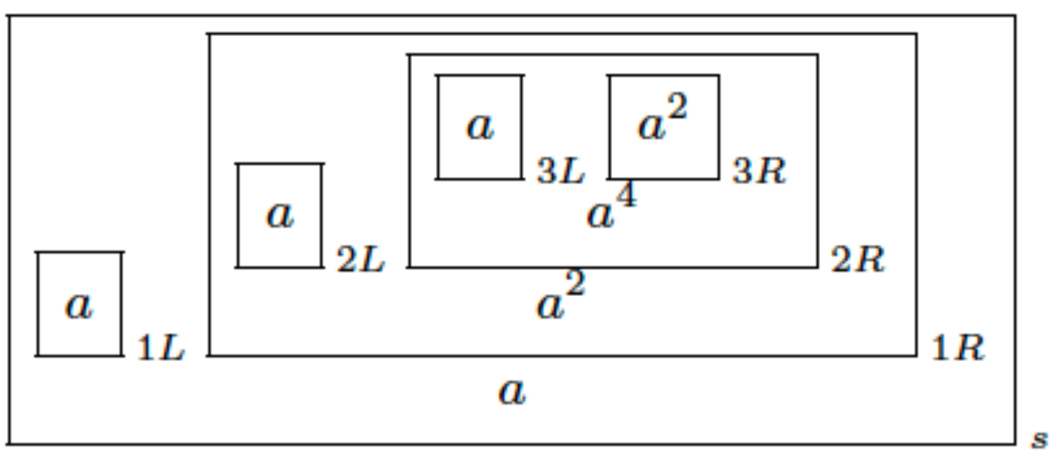
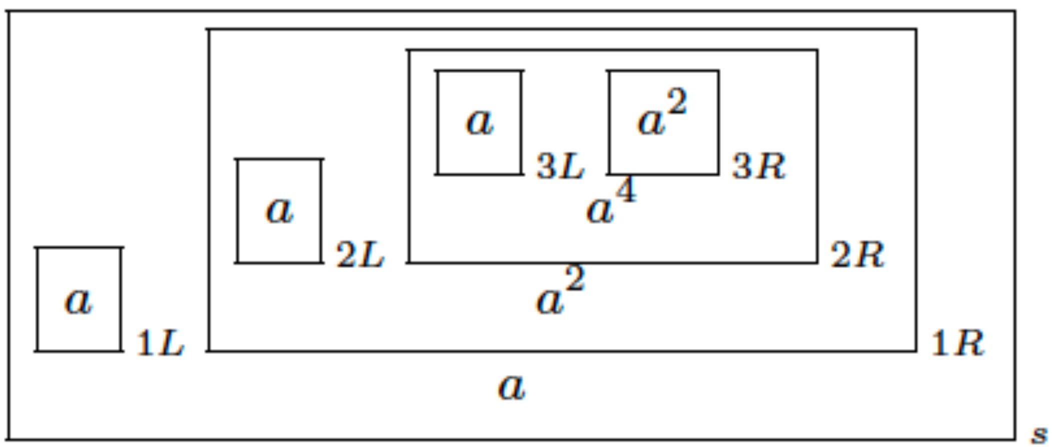
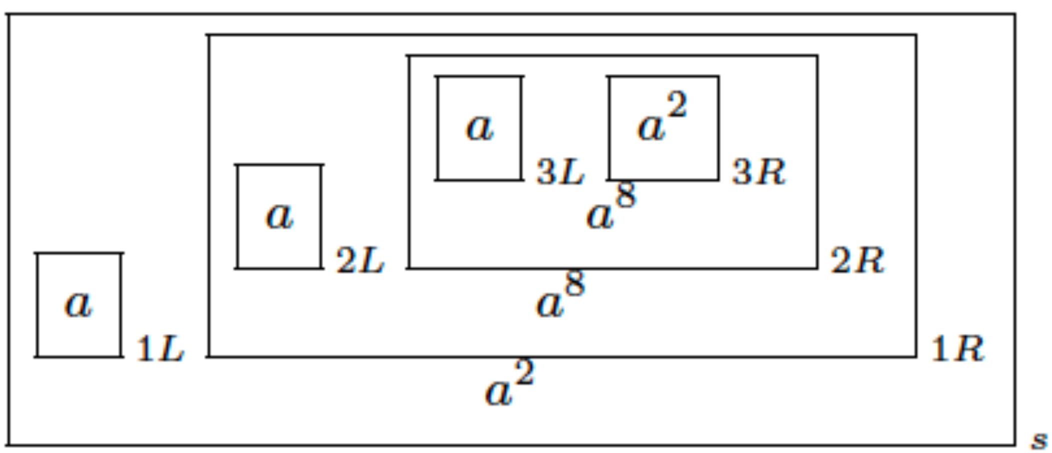


$3 : a \to a^2$ in $2R$
$2 : a \to a^4$ in $1R$
$1 : a \to a^2$ in s

For example

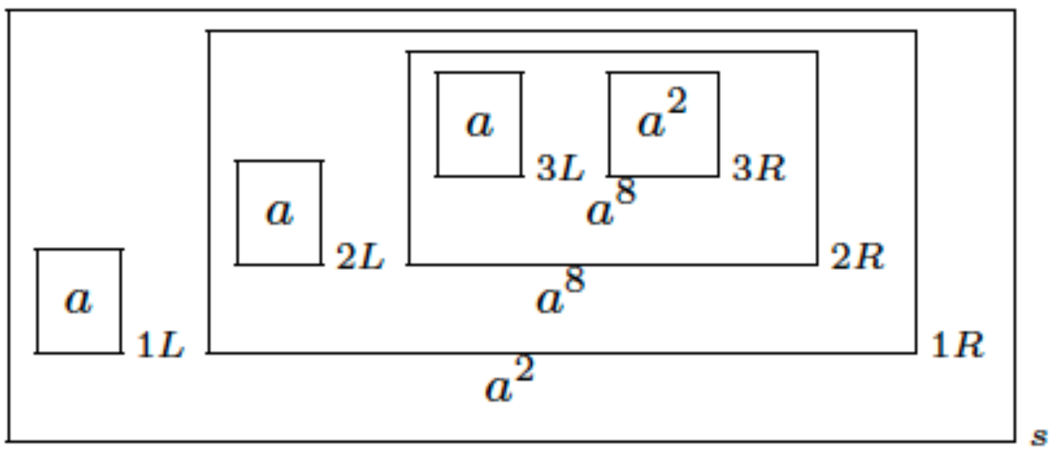


$$3 : a \rightarrow a^2 \text{ in } 2R$$
$$2 : a \rightarrow a^4 \text{ in } 1R$$
$$1 : a \rightarrow a^2 \text{ in } s$$
$$\Rightarrow$$



$$3 : a \rightarrow a^2 \text{ in } 2R$$
$$2 : a \rightarrow a^8 \text{ in } 1R$$
$$1 : a \rightarrow a^8 \text{ in } s$$

For example



$3 : a \rightarrow a^2$ in $2R$
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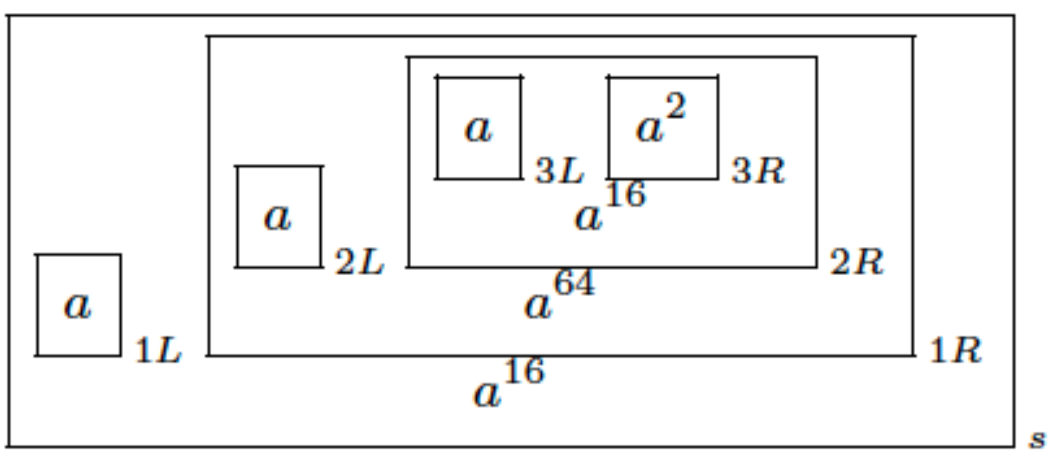
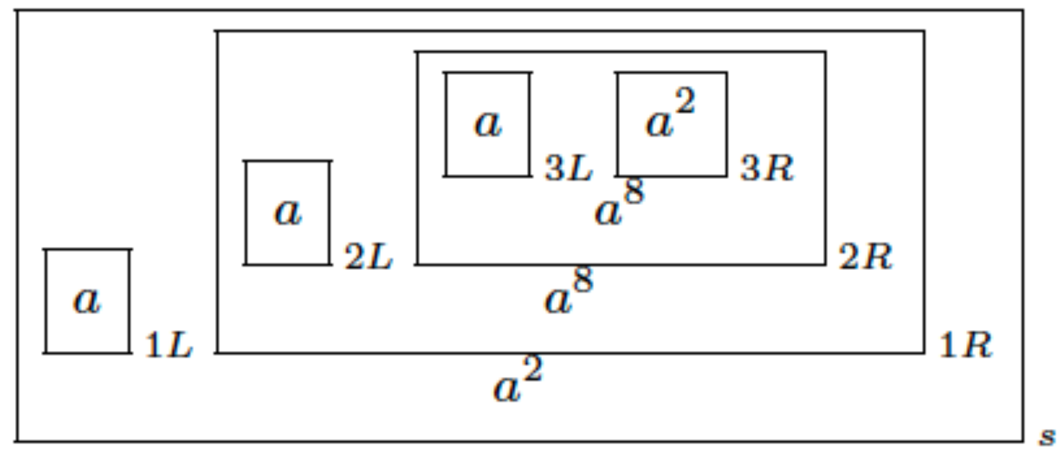
$1 : a \rightarrow a^8$ in s
$\Rightarrow$

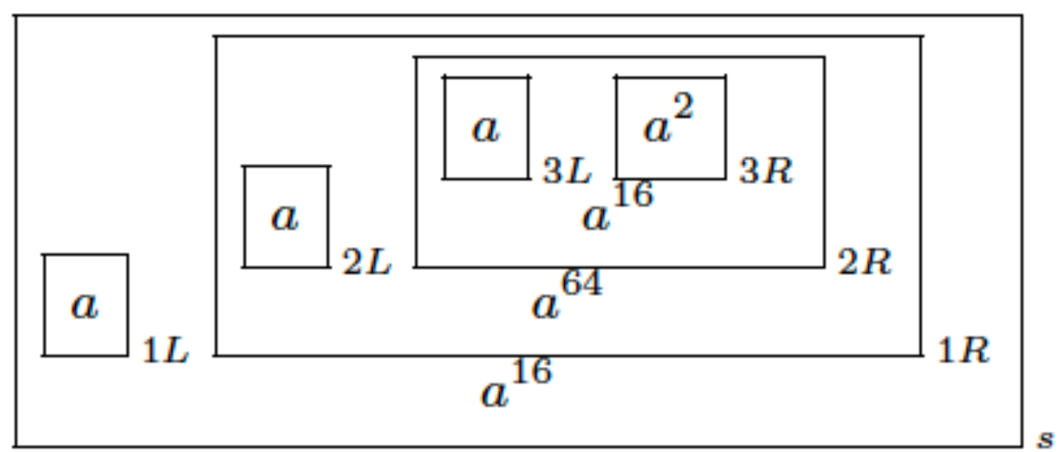


$3 : a \rightarrow a^2$ in $2R$
$2 : a \rightarrow a^{16}$ in $1R$
$1 : a \rightarrow a^{64}$ in s
$\Rightarrow \cdots$

For example



$3 : a \rightarrow a^2 \text{ in } 2R$
$2 : a \rightarrow a^8 \text{ in } 1R$
$1 : a \rightarrow a^8 \text{ in } s$
$\Rightarrow$



$3 : a \rightarrow a^2 \text{ in } 2R$
$2 : a \rightarrow a^{16} \text{ in } 1R$
$1 : a \rightarrow a^{64} \text{ in } s$
$\Rightarrow \cdots$

$$(2, 2^n, 2^{n(n-1)/2}, 2^{n(n-1)(n-2)/6})$$

Notation

$$NOP_k(polym_{+d}(coo), tar),\ \ PsOP_k(polym_{+d}(coo), tar),$$
$$fDOP_k(polym_{+d}(coo), tar).$$

- Numbers, vectors, functions
- The number of regions
- Rule “disabling”
- Determinism
- Cooperating, non-cooperating rules
- Target indicators
- ….

Theorem 2. *There exist*

- *A strongly universal P system from* $OP_{47}(polym_{-d}(coo))$;
- *A P system* $\Pi_1 \in DOP_7(polym_{-d}(ncoo))$ *with a superexponential growth;*
- *A P system* $\Pi_2 \in OP_{13}(polym_{-d}(ncoo), tar)$ *such that* $N(\Pi_2) = \{n! \cdot n^k \mid n \geq 1,\ k \geq 0\}$ *and the time complexity of generating* $n! \cdot n^k$ *is* $n + k + 1$;
- *A P system* $\Pi_3 \in OP_9(polym_{-d}(coo), tar)$ *such that* $N(\Pi_3) = \{n! \mid n \geq 1\}$ *and the time complexity of generating* $n!$ *is* $n + 1$;
- *A P system* $\Pi_4 \in OP_{15}(polym_{-d}(ncoo), tar)$ *such that* $N(\Pi_4) = \{2^{2^n} \mid n \geq 0\}$ *and the time complexity of generating* 2^{2^n} *is* $3n + 2$;
- *A P system* $\Pi_5' \in DOP_*(polym_{-d}(coo), tar)$ *such that* $f(\Pi_5) = (n \longrightarrow 2^{2^n})$ *and the time complexity of computing* $n \longrightarrow 2^{2^n}$ *is* $O(n)$;
- *A P system* $\Pi_6 \in DOP_*(polym_{-d}(coo), tar)$ *such that* $N_d(\Pi_6) = \{n! \mid n \geq 1\}$ *and the complexity of deciding any number* k, $k \leq n!$ *does not exceed* $4n$.

Moreover, polymorphic P systems can grow faster than any non-polymorphic P systems, whereas even non-cooperative polymorphic P systems with targets can grow faster than any polymorphic P systems without targets.

Interesting problem

- To charcaterize the power of the **non-cooperating** variants

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Variants of non-cooperativity

- **Strong non-cooperative** systems: left membranes contain at most one symbol
- **Weak non-cooperative systems**: all rules which are actually applied have one symbol on their left-hand side

Theorem 2. $NOP_*(polym_{+d}(ncoo_w)) = NOP_*(polym_{+d}(ncoo_s))$.

Rule disabling doesn't matter

Proposition 1. $NOP_*(polym_{-d}(ncoo)) = NOP_*(polym_{+d}(ncoo))$.

Left polymorphism

- In general, as a consequence of certain lemmas: left membranes with “invariable rules” are sufficient
- Left polymorphic systems are more powerful than conventional transition P systems, but they cannot generate everything:

Proposition 2. $L_{2^n} = \{2^n \mid n \in \mathbb{N}\} \in NOP_*(lpolym(ncoo))$.

Proposition 3. $L_{n!} = \{n! \mid n \in \mathbb{N}\} \notin NOP_*(lpolym_{+d}(ncoo))$.

What is left polymorphism?

A depth-based hierarchy

Theorem 4. $L_{d+1} = \left\{ 2^{\binom{n}{d-1}} \,\middle|\, n \in \mathbb{N}, n > d \right\} \notin NOP_*^d(polym(ncoo)),\ d > 1.$

Corollary 3. $NOP_*^d(polym(ncoo)) \subsetneq NOP_*^{d+1}(polym(ncoo)).$

A depth-based hierarchy

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As a consequence for left polymorphic systems:

Corollary 4. $NOP_*(lpolym(ncoo)) \subsetneq NOP_*(polym(ncoo)).$

(since the left membranes with depth 3 are sufficient to reach their maximal power)

Right polymorphism and general polymorphism

Proposition 4. $L'_{2^n} = \{2^n \mid n \in \mathbb{N}, n > 2\} \in NOP_*(rpolym(ncoo))$.

Theorem 3. $L_{n!} = \{n! \mid n \in \mathbb{N}\} \notin NOP_*(polym(ncoo))$.

Some open problems

- The relationship of the right polymorphic and the general variants
- Upper bounds on the power of certain variants
- Polymorphic systems seem to lie between systems with static membrane structures and systems with dynamic membrane structures (active membranes)

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For some initial ideas, see the next presentation…