Gandy-Păun-Rozenberg machines

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Mathematics is a powerful tool that helps people achieve new goals as well as understand what is impossible to do.
Mark Burgin

Foundations

The paper *Logically Possible Machines* by Eric Steinhart (*Mind and Machines* 12 (2002), pp. 259–280) contains a discussion of logical foundations of computation theory including quantum computation which gives rise to the following family of questions:

(?) what is it an $\mathcal{X}$ possible machine?

for $\mathcal{X} \in \{$set-theoretically, discrete topologically, continuous topologically, geometrically, biologically inspired, physically, cognitive and intelligent$\}$.

Gandy's machine

We point out here that Robin Gandy's machines (cf. Gandy's paper *Church Thesis and Principles of Mechanisms* in: *The Kleene Symposium*, ed. J. Barwise et al., 1980, pp. 123–148) yield some answer to (?) for $\mathcal{X} \equiv$ set-theoretically in discrete case. The physically possible machines are discussed in the papers about physical limitations of computing devices by Scott Aaronson, Jacob Bekenstein, Charles H. Bennett, Rolf Landaurer, Stockmeyer and Meyer, among others.

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The paper *Can Cognitive and Intelligent Systems Outperform Turing Machines* by Jiři Wiedermann inspired to formulate (?) for $\mathcal{X} \equiv$ cognitive and intelligent.

G–P–R machine

An idea of a Gandy–Păun–Rozenberg machine, briefly G–P–R machine, is aimed to provide an answer to (?) for
$\mathcal{X} \equiv$ set-theoretically,
$\mathcal{X} \equiv$ discrete topologically, and
$\mathcal{X} \equiv$ biologically inspired.

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- Gandy's machines,
- P systems due to Gheorghe Păun (cf. *Membrane Computing. An Introduction*, Berlin 2002),
- parallel rewriting systems of graphs investigated by Grzegorz Rozenberg himself with scientists cooperating with him, among others, in preparation of many volume *Handbook of graph grammars and computing by graph transformation*.

Core

The core of a G–P–R machine is a finite set of rewriting rules for certain finite directed labelled graphs, where these graphs are instantenous descriptions for the computation process realized by the machine.

Parallelism

The conflictless parallel (simultaneous) application of the rewriting rules of a G–P–R machine is realized in Gandy's machine mode (according to Local Causality Principle), where (local) maximality of "causal neighbourhoods" replaces (global) maximality of, e.g. conflictless set of evolution rules applied simultaneously to a membrane structure which appears during the evolution process generated by a P system. Therefore one can construct a Gandy's machine from a G–P–R machine in an immediate way.

NP problems

The NP complete problems can be solved by G–P–R machines in a polynomial time (but with an exponential number of indecomposable processors), where one constructs the G–P–R machines solving these problems in a polynomial time in a similar way to (families of) P systems solving these problems also in a polynomial time (cf. the pioneering Păun's paper *P systems with active membranes: Attacking NP complete problems*, Journal of Automata Languages and Combinatorics 6 (2000), pp. 75–90).

Future investigations

Randomized G–P–R machines for solving NP problems in a polynomial time with subexponential number of indecomposable processors are forthcoming.

Future investigations

An extension of G–P–R machines to the case of cellular automata can be done by adopting the idea of cellular hypergraph rewriting introduced by Peter Hartmann in his paper *Parallel Replacement Systems on Geometric Hypergraphs: A Mathematical Tool for Handling Dynamic Geometric Sceneries* in: *Proceedings of PARCELLA'94 Workshop*, Akademie Verlag, Potsdam 1994, pp. 81–90.