On the Semantics of Annihilation Rules in Membrane Computing

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Background

▷ $AM^0_{-d,+ne}$

(a) $[\, a \to u \,]_h$

(b) $a\,[\ \ ]_h \to [\, b \,]_h$

(c) $[\, a \,]_h \to b\,[\ \ ]_h$

(d) $[\, a \,]_h \to [\, b \,]_h\ [\, c \,]_h$

(e) $[\,[\ \ ]_{h_1}[\ \ ]_{h_2}\,]_{h_0} \to [\,[\ \ ]_{h_1}\,]_{h_0}\ [\,[\ \ ]_{h_2}\,]_{h_0}$

$$\mathbf{PMC}_{\mathcal{AM}^0_{-d,+ne}} = \mathbf{P}$$

[1]

[1] Gutiérrez-Naranjo, M.A., Pérez-Jiménez, M.J., Riscos-Núñez, A., Romero-Campero, F.J.: On the power of dissolution in P systems with active membranes. In: Freund, R., Păun, Gh., Rozenberg, G., Salomaa, A. (eds.) Workshop on Membrane Computing. Lecture Notes in Computer Science, vol. 3850, pp. 224–240. Springer, Berlin Heidelberg (2005)

Background

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  (d) $[\, a \,]_h \to [\, b \,]_h\ [\, c \,]_h$

  (e) $[\,[\ ]_{h_1}[\ ]_{h_2}\,]_{h_0} \to [\,[\ ]_{h_1}\,]_{h_0}\ [\,[\ ]_{h_2}\,]_{h_0}$

  (f) $[\, a\bar{a} \,] \to \lambda$

$$\mathbf{PMC}_{\mathcal{AM}^0_{-d,+ne,+ant}} = \mathbf{NP}$$

[2]

[2] Díaz-Pernil, D., Peña-Cantillana, F., Alhazov, A., Freund, R., Gutiérrez-Naranjo, M.A.: Antimatter as a frontier of tractability in membrane computing. Fundamenta Informaticae 134, 83–96 (2014)

Frontier

- **So, antimatter (and annihilation rules) is a new frontier of tractability**
- **It is a nice result of the type**
- **If $*$ is considered, the model solves NP,**
  If $*$ is not considered, it solves P
- **. . . where $*$ is one of the many frontiers studied en the literature!**

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- **More exactly ...**
- **... where $*$ is one of the many syntactical frontiers studied en the literature!**

Frontier

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- **More exactly . . .**
- **. . . where $*$ is one of the many syntactical frontiers studied en the literature!**
- **But, can the semantics be used as a frontier of the tractability?**

- **In other words . . .**
- **In the proof of**

$$\textbf{PMC}_{\mathcal{AM}^0_{-d,+ne,+ant}} = \textbf{NP}$$

- **annihilation rules** $[\, a\bar{a}\,] \to \lambda$ **use priority on the other rules . . .**
- **. . . with a physical inspiration:** *An electron has no chance if it meets a positron!*
- **What happens if priority is removed?**
- **Is the model still able to solve NP?**

▷ **Is the model still able to solve NP?**

No

Formally

$$\mathbf{PMC}_{\mathcal{AM}^0_{-d,+ne,+ant}NoPri} = \mathbf{P}$$

- **where**

$$\mathcal{AM}^0_{-d,+ne,+ant}NoPri$$

is syntactically the same P system model as

$$\mathcal{AM}^0_{-d,+ne,+ant}$$

but, with a semantic difference: Annihilation rules have no priority on the other rules.

Hint of the proof

$$\mathbf{PMC}_{\mathcal{AM}^0_{-d,+ne,+ant\,NoPri}} = \mathbf{P}$$

- **Two proofs:**
- **The first one (trivial)**

$$\mathbf{P} = \mathbf{PMC}_{\mathcal{AM}^0_{-d,+ne}} \subseteq \mathbf{PMC}_{\mathcal{AM}^0_{-d,+ne,+ant\,NoPri}}$$

- **The interesting one**

$$\mathbf{PMC}_{\mathcal{AM}^0_{-d,+ne,+ant\,NoPri}} \subseteq \mathbf{PMC}^*_{\mathcal{AM}^0_{-d,+ne}} = \mathbf{P}$$

Hint of the proof

- **The interesting one**

$$\mathbf{PMC}_{\mathcal{AM}^0_{-d,+ne,+antNoPri}} \subseteq \mathbf{PMC}^*_{\mathcal{AM}^0_{-d,+ne}} = \mathbf{P}$$

- **Let us consider a decision problem**

$$\theta \in \mathbf{PMC}_{\mathcal{AM}^0_{-d,+ne,+antNoPri}}$$

- **We will prove that**

$$\theta \in \mathbf{PMC}^*_{\mathcal{AM}^0_{-d,+ne}}$$

Hint of the proof

▷ **The interesting one**

$$\mathbf{PMC}_{\mathcal{AM}^0_{-d,+ne,+antNoPri}} \subseteq \mathbf{PMC}^*_{\mathcal{AM}^0_{-d,+ne}} = \mathbf{P}$$

▷ **If** $\theta \in \mathbf{PMC}_{\mathcal{AM}^0_{-d,+ne,+antNoPri}}$ **...**

▷ **... there exists a pair of functions** (cod, s) **and a family of P systems** $\{\Pi_i\}_{i\in\mathbb{N}}$ **in** $\mathcal{AM}^0_{-d,+ne,+antNoPri}$ **verifying that for each instance** u **of the problem** θ

- **All computation of** $\{\Pi_{s(u)} + cod(u)\}$ **halt;**
- **All computation send out** YES **or** NO **(but no both) in the last step of computation.**

Hint of the proof

- **The interesting one**

$$\mathbf{PMC}_{\mathcal{AM}^0_{-d,+ne,+antNoPri}} \subseteq \mathbf{PMC}^*_{\mathcal{AM}^0_{-d,+ne}} = \mathbf{P}$$

- **Let us consider one instance** $u \in \theta$
- **Let** $\mathcal{C} = \{C_0, \dots, C_n\}$ **be one of such halting configurations of** $\{\Pi_{s(u)} + cod(u)\}$
- **Let us suppose that the answer is** YES
- **It is clear that there exist an object** a_1 **and a rule**

$$r_1 \equiv [\, a_1 \,]_{skin} \rightarrow YES\, [\,]_{skin}$$

which has been applied in the last step of computation.

Hint of the proof

- **The interesting one**

$$\mathbf{PMC}_{\mathcal{AM}^0_{-d,+ne,+antNoPri}} \subseteq \mathbf{PMC}^*_{\mathcal{AM}^0_{-d,+ne}} = \mathbf{P}$$

- **Let r_2 be one of the rules which have produced the occurrence of a_1 in the skin.**
- **Notice that at least one such r_2 must exist, but it cannot be unique!**
- **We choose one of such r_2**

Hint of the proof

- **The interesting one**

$$\mathbf{PMC}_{\mathcal{AM}^0_{-d,+ne,+antNoPri}} \subseteq \mathbf{PMC}^*_{\mathcal{AM}^0_{-d,+ne}} = \mathbf{P}$$

- **Such r_2 is triggered by the occurrence of an object a_2 in a membrane with label h_2**
- **Obviously, r_2 cannot be an annihilation rule!**
- **We go back with the reasoning and a_2 appears in the membrane with label h_2 by the application of a rule r_3 and so on**

Hint of the proof

- **The interesting one**

$$\mathbf{PMC}_{\mathcal{AM}^0_{-d,+ne,+antNoPri}} \subseteq \mathbf{PMC}^*_{\mathcal{AM}^0_{-d,+ne}} = \mathbf{P}$$

- **Finally we have a chain**

$$(YES, env) \xleftarrow{r_1} (a_1, skin) \xleftarrow{r_2} (a_2, h_2) \xleftarrow{r_3} \ldots \xleftarrow{r_k} (a_k, h_k)$$

- **where $k \leq n$ and a_k appears in a membrane with label h_k in the initial configuration.**

Hint of the proof

- **The interesting one**

$$\textbf{PMC}_{\mathcal{AM}^0_{-d,+ne,+ant\,NoPri}} \subseteq \textbf{PMC}^*_{\mathcal{AM}^0_{-d,+ne}} = \textbf{P}$$

- **Finally, for the instance $u \in \theta$, let us consider the P system Π'_u with only one membrane with label s and only one object (a_k, h_k) in the initial configuration**
- **The set of rules is**
  - $[(a_i, h_i) \rightarrow (a_{i-1}, h_{i-1})]_s$ **for each** $i \in \{3, \ldots, k-1\}$
  - $[(a_2, h_2) \rightarrow (a_1, skin)]_s$
  - $[(a_1, skin)]_s \rightarrow YES\,[\,]_s$

Hint of the proof

▷ **The interesting one**

$$\mathbf{PMC}_{\mathcal{AM}^0_{-d,+ne,+antNoPri}} \subseteq \mathbf{PMC}^*_{\mathcal{AM}^0_{-d,+ne}} = \mathbf{P}$$

▷ **So,** $\Pi'_u \in \mathcal{AM}^0_{-d,+ne}$

▷ **It has been built for computing an answer for** $u \in \theta$

▷ **It is deterministic. It has only one halting computation which sends out** YES **in the last step of computation. Therefore. . .**

$$\theta \in \mathbf{PMC}^*_{\mathcal{AM}^0_{-d,+ne}} = \mathbf{P}$$

Finally

▷ **That's all . . .**

Thanks!

- Questions?
- Comments?
- Suggestions?