Membrane computing as a framework for modelling the dynamics of complex systems in ecology

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Topics

- Membrane computing as a framework for computational modelling
- Applications to complex systems of interest (ecology)

Membrane computing

- A branch of natural computing inspired by
 - the structure and function of living cells or
 - the cooperation of cell populations in tissues, organs, and neural networks



Cell membrane, source: Wikipedia

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Membrane (P) systems

Definition

A skeleton of an extended P system with active membranes of degree $q \ge 1$ is a tuple

$$\Pi = (\Gamma, \mu, \mathcal{R}),$$

where

- Γ is an alphabet (the working alphabet),
- μ is a membrane structure consisting of q membranes labelled by 1,..., q in a one-to-one manner (label 1 denotes the skin membrane),
- \bullet electrical charges out of the set $\{0,+,-\}$ are also associated with the membranes, and
- \mathcal{R} is a finite set of evolution rules of the form $r : u[v]_i^{\alpha} \to u'[v']_i^{\alpha'}$, where u, v, u', v' are multisets over Γ , $i \in \{1, \ldots, q\}$, and $\alpha, \alpha' \in \{0, +, -\}$.

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Membrane (P) systems

Remark

- A skeleton of an extended P system can be viewed as a set of (polarized) membranes organized in a hierarchical structure μ and labelled by 1,..., q.
- At the beginning, all membranes of μ are neutral.

Recent advancements

- tackling computationally hard problems [Garey and Johnson, 1979, Papadimitriou, 1994]
 - NP-complete [Díaz-Pernil et al., 2008, Leporati et al., 2007, Păun, 2001, Pérez-Jiménez and Riscos-Núñez, 2004, Pérez-Jiménez and Riscos-Núñez, 2005] or
 - PSPACE-complete [Song and Zeng, 2021b] problems

Definition

A multienvironment P system of degree (q, m, n) and with T time units is

$$\Pi = (G, \Gamma, \Sigma, \Phi, T, n = \sum_{j=1}^{m} n_j, \{\Pi_{k,j} \mid 1 \le k \le n_j, 1 \le j \le m\}, \{(f_j, E_j) \mid 1 \le j \le m\}, \mathcal{R}_E\}$$

- G = (V, S) is a directed graph with $m \ge 1$ nodes, and $V = \{e_1, \ldots, e_m\}.$
- Γ, Σ and Φ are alphabets such that $\Sigma \subseteq \Gamma$, and $\Gamma \cap \Phi = \emptyset$.
- T, $n \in \mathbb{N}$ and $T, n \ge 1$, with $n = \sum_{j=1}^{m} n_j$ and $n_j \ge 0$.

Definition

- $\Pi_{k,j}$ is defined as $\Pi_{k,j} = (\Gamma, \mu, \mathcal{M}_{1,j}^k, \dots, \mathcal{M}_{q,j}^k, \mathcal{R}_j, i_{in})$ for each k, j $(1 \le k \le n_j, 1 \le j \le m)$, where
 - * μ is a rooted tree with $q \ge 1$ nodes labelled by elements of $\{1, \ldots, q\} \times \{0, +, -\}.$
 - * $\mathcal{M}_{i,i}^k$ is a multiset over Γ for each i, $1 \le i \le q$.
 - * \mathcal{R}_j is a finite set of rules of the form $r : u[v]_i^{\alpha} \xrightarrow{P} u'[v']_i^{\alpha'}$, where u, v, u', v' are multisets over Γ , $1 \le i \le q$, $\alpha, \alpha' \in \{0, +, -\}$, and p is a computable function whose domain is $\{0, \ldots, T\}$, depending on the environment e_j .

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* i_{in} is a node of μ .

Definition

- For each j, $1 \le j \le m$, $f_j \in \Phi$ and E_j is a multiset over Σ .
- \mathcal{R}_E is a finite set of communication rules between environments of the following forms:

$$(x)_{e_j} \xrightarrow{p_1} (y_1)_{e_{j_1}} \cdots (y_h)_{e_{j_h}}, \quad (\Pi_{k,j})_{e_j} \xrightarrow{p_2} (\Pi_{k,j})_{e_{j_1}},$$

$$\{f\}(u)_{e_j} \xrightarrow{p_3} (v)_{e_{j_1}}, \quad \{f\}(u,f)_{e_j} \xrightarrow{p_4} (v,g)_{e_j}$$

where $x, y_1, \ldots, y_h \in \Sigma$, $(e_j, e_{j_i}) \in S$, $1 \le j \le m$, $1 \le i \le h$, $1 \le k \le n$, $f, g \in \Phi$, u, v are multisets over Γ , and p_1, p_2, p_3, p_4 are computable functions whose domains are $\{0, \ldots, T\}$.

Remark

A multienvironment P system of degree (q, m, n) with T time units consists of

- m environments e_j connected by the arcs of a directed graph G,
- n = n₁ + ··· + n_m P systems of order q, all with the same skeleton:
 Π_{1,1}...Π_{n1,1} (in environment e₁)
 Π_{1,2}...Π_{n2,2} (in environment e₂)
 ...
 ...
 Π_{1,m}...Π_{nm,m} (in environment e_m)
- the Γ working alphabet, the Σ alphabet of objects of the environments and the Φ alphabet of the flags, and
- a set of communication rules that allow objects to move between different environments.

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Remark

Initially, each environment e_j

- has a unique flag f_j,
- contains a multiset of objects E_j from the alphabet Σ of the environments,
- contains the P systems $\Pi_{1,j}, \ldots, \Pi_{n_j,j}$, all with the same skeleton and with rules associated with computable functions over $\{0, \ldots, T\}$ that depend on e_j .

Remark

At any given moment, an environment e_i

- has a unique flag,
- \bullet contains objects from the Σ alphabet of environments, and
- includes some P systems.

Configurations

Remark

- $u[v]_i^{\alpha} \xrightarrow{p} u'[v']_i^{\alpha'}$: the multisets u, v produce u', v', respectively, and the charge of membrane *i* changes from α to α' .
- $(x)_{e_j} \xrightarrow{p_1} (y_1)_{e_{j_1}} \cdots (y_h)_{e_{j_h}}$: object x moves from environment e_j to environments $e_{j_1} \cdots e_{j_h}$, transforming into $y_1 \cdots y_h$ in the respective environments.
- $(\Pi_{k,j})_{e_j} \xrightarrow{p_2} (\Pi_{k,j})_{e_{j'}}$: the system $\Pi_{k,j}$ moves from e_j to $e_{j'}$.

Configurations

Remark

- {f}(u)_{ej} → (v)_{ej1}: multiset u produces multiset v, while preserving the flag f in ej. Since the flag f is not consumed, this rule can be applied multiple times in a single step.
- {f}(u, f)_{ej} → (v, g)_{ej}: multiset u produces multiset v and flag f produces flag g. Given that the flag f is consumed and that each environment has a unique flag, this rule can be applied only once, at most, in each step.
- The functions p(t), $p_1(t)$, $p_2(t)$, $p_3(t)$ and $p_4(t)$ determine the units of the applicable rules at time t.

Transition step from one configuration to another: a maximal multiset of applicable rules will be applied simultaneously, selected according to the probabilities assigned to them, and all occurrences of the left-hand side of the rules are consumed.

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Stochastic approach

- The alphabet of the flags is empty.
- The *n* systems $\Pi_{k,j}$ are independent of the index *j* (hence Π_k , $1 \le k \le n$ is simply written).
- Initially, the *n* systems Π_k are randomly distributed among the *m* environments of the system.
- The computable functions associated with the rules are propensities: they depend not on the environment, but on time.

Inference engine: multicompartmental Gillespie's algorithm and deterministic waiting time algorithm.

Stochastic approach



Probabilistic approach

- The number n of Π_{k,j} systems is at most the number m of environments, i.e. n ≤ m.
- The functions p associated with the rules $r : u[v]_i^{\alpha} \xrightarrow{p} u'[v']_i^{\alpha'}$ of $\prod_{k,j}$ are probability functions: at each time instant, the sum of the functions associated with the rules of \mathcal{R}_j whose LHS is $u[v]_i^{\alpha}$, sum to 1.
- The functions p₁ associated with the environmental rules
 (x)_{ej} → (y₁)_{ej₁} ··· (y_h)_{ej_h} are probability functions: at each time
 instant, the sum of the functions associated with the rules whose LHS
 is (x)_{ej}, sum to 1.

Probabilistic approach

- The functions p_2 associated with the environmental rules $(\prod_{k,j})_{e_j} \xrightarrow{p_2} (\prod_{k,j})_{e_{j'}}$ are constants and equal to 0.
- The functions p_3, p_4 associated with the environmental rules $\{f\}(u)_{e_j} \xrightarrow{p_3} (v)_{e_{j_1}}$ and $\{f\}(u, f)_{e_j} \xrightarrow{p_4} (v, g)_{e_j}$ are probability functions.
- There are no rules $u[v]_i^{\alpha} \xrightarrow{p} u'[v']_i^{\alpha'}$ in the skin membrane of the P systems and the rules of the environments $(x)_{e_j} \xrightarrow{p_1} (y_1)_{e_{j_1}} \cdots (y_h)_{e_{j_h}}$ such that $x \in u$ (no competition).
- Initially, each environment e_j contains at most one system $\Pi_{k,j}$.

Inference engine: BBB, DNDP and DCBA algorithms.

Probabilistic approach



Definition

Multicompartmental P systems are multienvironment P systems with stochastic approach:

$$\Pi = (G, \Gamma, \Sigma, \Phi, T, n, \{\Pi_k \mid 1 \le k \le n\}, \{E_j \mid 1 \le j \le m\}, \mathcal{R}_E)$$

- G = (V, S) is a directed graph with $m \ge 1$ nodes, $V = \{e_1, \ldots, e_m\}$.
- Γ and Σ are alphabets such that $\Sigma \subsetneq \Gamma$. Moreover, $\Phi = \emptyset$.
- $T, n \in \mathbb{N}$ and $T, n \geq 1$.

- The system Π_k is $(\Gamma, \mu, \mathcal{M}^1_k, \dots, \mathcal{M}^q_k, \mathcal{R}, i_{in})$ for each $k, 1 \le k \le n$,
 - * μ is a rooted tree with $q \ge 1$ nodes labelled by elements of $\{1, \ldots, q\} \times \{0, +, -\}.$
 - * \mathcal{M}_k^i is a multiset over Γ for each $i, 1 \leq i \leq q$.
 - * \mathcal{R} is a finite set of rules of the type: $u[v]_i^{\alpha} \xrightarrow{p} u'[v']_i^{\alpha'}$, where u, v, u', v' are multisets over Γ , $1 \leq i \leq q$, $\alpha, \alpha' \in \{0, +, -\}$, and p is a computable function whose domain is $\{0, \ldots, T\}$ that represents the propensity.
 - * i_{in} is a node of μ .

- E_j is a multiset over Σ for each j, $1 \le j \le m$.
- \mathcal{R}_E is a finite set of communication rules between environments of the type:
 - * $(x)_{e_j} \xrightarrow{p_1} (y_1)_{e_{j_1}} \cdots (y_h)_{e_{j_h}}, (\Pi_k)_{e_j} \xrightarrow{p_2} (\Pi_k)_{e_{j_1}}, \text{ where } x, y_1, \ldots, y_h \in \Sigma, (e_j, e_{j_i}) \in S, \ 1 \le j \le m, \ 1 \le i \le h, \ 1 \le k \le n, \ \text{and} \ p_1, p_2 \ \text{are propensities whose domain is } \{0, \ldots, T\} \ \text{(they do not depend on the environment } e_j).$

Remark

- A multicompartimental P system of degree (q, m, n) with T time units consists of
 - m environments connected by arcs of a directed graph G.
 - ▶ *n* P systems of degree q, all Π_k having the same skeleton.
- Initially, the $n \prod_k$ systems are randomly distributed among the m environments of the system.
- At each moment, an environment e_j can contain only objects from Σ and some P systems.

Remark

- $u[v]_i^{\alpha} \xrightarrow{p} u'[v']_i^{\alpha'}$: the multisets u, v produce u', v', respectively, and the charge of membrane *i* changes from α to α' .
- $(x)_{e_j} \xrightarrow{p_1} (y_1)_{e_{j_1}} \cdots (y_h)_{e_{j_h}}$: object x moves from environment e_j to environments $e_{j_1} \cdots e_{j_h}$, transforming into $y_1 \cdots y_h$ in the respective environments.
- $(\Pi_{k,j})_{e_j} \xrightarrow{p_2} (\Pi_{k,j})_{e_{j'}}$: the system $\Pi_{k,j}$ moves from e_j to $e_{j'}$.
- The functions p(t), p₁(t) and p₂(t) determine the units of some applicable rules at time t (propensities: they only depend on the multiplicities of y objects at that instant).

Inference engine: multicompartmental Gillespie's algorithm and deterministic waiting time algorithm.

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Multicompartmental Gillespie's algorithm

- the Gillespie's algorithm offers an exact method for the stochastic simulation of biochemical reactions
- the multicompartmental Gillespie's algorithm [Cheruku et al., 2007] is an extension of the classical Gillespie's algorithm
- unlike the original version, this method considers the existence of multiple disjoint compartments that represent different regions where biochemical reactions occur

Definition

Population Dynamics P systems (PDP) are multienvironment P systems with probabilistic approach:

 $\Pi = (G, \Gamma, \Sigma, \Phi, T, n, \{\Pi_k \mid 1 \le k \le n\}, \{E_j \mid 1 \le j \le m\}, \mathcal{R}_E)$

- G = (V, S) is a directed graph with $m \ge 1$ nodes, $V = \{e_1, \ldots, e_m\}$.
- Σ and Γ are alphabets such that $\Gamma \subsetneq \Sigma$. Furthermore, $\Phi = \emptyset$.
- $T, n \in \mathbb{N}$ and $T, n \geq 1$.

- The system Π_k is $(\Gamma, \mu, \mathcal{M}_1, \dots, \mathcal{M}_q, \mathcal{R}, i_{in})$ for each $k, 1 \leq k \leq n$, where
 - * μ is a rooted tree with $q \ge 1$ nodes labelled by elements of $\{1, \ldots, q\} \times \{0, +, -\}.$
 - * \mathcal{M}_i is a multiset over Γ for each $i, 1 \leq i \leq q$.
 - * \mathcal{R} is a finite set of rules of the type $u[v]_i^{\alpha} \xrightarrow{p} u'[v']_i^{\alpha'}$, where u, v, u'and v' are multisets over Γ , $1 \leq i \leq q$, $\alpha, \alpha' \in \{0, +, -\}$ and p is a probability function with domain $\{0, \ldots, T\}$. At each time instant, the sum of the probabilities associated with the rules with the same LHS is 1.
 - * i_{in} is a node of μ .

- E_j is a multiset over Σ for each j, $1 \le j \le m$.
- *R_E* is a finite set of environmental rules of the type
 (x)_{ej} → (y₁)_{ej1} ··· (y_h)_{ejh}, where x, y₁,..., y_h ∈ Σ, (e_j, e_{ji}) ∈ S,
 1 ≤ j ≤ m, 1 ≤ i ≤ h, u, v are multisets over Γ and p₁ is a probability
 function with domain {0,..., T}. At each time instant, the sum of
 the probabilities associated with the rules with the same LHS is 1.

- There are no rules u[v]_i^α → u'[v']_i^{α'} in the skin membrane of the P systems and no environmental rules (x)_{ej} → (y₁)_{ej1} ··· (y_h)_{ejh} such that x ∈ u (there is no competition between the rules of the skin membrane of the P system and the rules of the environment).
- Initially, each environment e_j contains exactly one system Π_k .

PDP systems

Remark

A PDP system of degree (q, m) consists of a set of m environments e_1, \ldots, e_m interconnected by the arcs of a directed graph G, each containing an ordinary P system.

- Each environment e_j can contain only symbols from the alphabet Σ and, additionally, a unique system $\Pi_k = (\Gamma, \mu, \mathcal{M}_1, \dots, \mathcal{M}_q, \mathcal{R}, i_{in})$ in such a way that
 - (a) The initial multisets of Π_k depend on the environment e_j .
 - ▶ (b) The probability functions associated with the rules of Π_k depend on the environment e_j .
- The probability functions associated with the rules of the environment e_i depend on that environment.

The semantics of PDP systems are implemented by ad hoc simulation algorithms.

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Probabilistic guarded P (PGP) systems

Definition

Probabilistic guarded P (PGP) systems are multienvironment P systems with probabilistic approach

$$\Pi = (G, \Gamma, \Sigma, \Phi, T, n, \{\Pi_{k,j} \mid 1 \le k \le n, 1 \le j \le m\}, \\ \{(f_j, E_j) \mid 1 \le j \le m\}, \mathcal{R}_E\}$$

- G = (V, S) is a directed graph with a single node.
- Σ and Φ are alphabets such that $\Sigma \cap \Phi = \emptyset$. Moreover, $\Gamma = \Sigma$.
- $T \ge 1, T \in \mathbb{N}$ and n = 0 (therefore there are no P systems of the type $\Pi_{k,j}$).
- $f_j \in \Phi$ and E_j is a multiset over Σ for each j, $1 \le j \le m$.

Probabilistic guarded P (PGP) systems

Definition

• \mathcal{R}_E is a finite set of environmental rules of the type

*
$$\{f\}(u)_{e_j} \xrightarrow{p_3} (v)_{e_{j_1}}$$
.

* $\{f\}(u,f)_{e_j} \xrightarrow{p_4} (v,g)_{e_j}$.

 $f, g \in \Phi, u, v$ are multisets over $\Sigma, 1 \leq j, j_1 \leq m$. $(e_j, e_{j_1}) \in S$ and p_3, p_4 are probability functions with domain $\{0, \ldots, T\}$. Furthermore, for each $f \in \Phi, u$ is a multiset over Σ .

- * There exists only one rule whose LHS is $\{f\}(u, f)_{e_j}$. In addition, the probability associated with that rule is 1.
- * The sum of the probabilities associated with the rules whose LHS is $\{f\}(u)_{e_i}$ is 1.

The degree of the above PGP system is m.

PGP systems

Remark

A PGP system of degree m can be briefly expressed through a tuple

$$\Pi = (\Sigma, \Phi, (f_1, E_1), \dots, (f_m, E_m), \mathcal{R}_E, p_{\mathcal{R}_E})$$

and be viewed as a set of *m* environments (which we will call cells in this case) interconnected by the arcs of a graph that is implicitly given in the structure by the set of rules \mathcal{R}_E .

- Each environment contains a unique flag (an element of Φ) at each time instant. We can think of the flags as being on the environments/cells. Initially, the flag of the environment e_j is f_j.
- Each environment contains a finite multiset of objects over Σ. We can imagine that the objects are in/inside the environments/cells. Initially, the multiset of objects of the environment e_i is M_i.

PGP systems

Remark

- There exists a finite set of environment rules that are of the form:
 - $\{f\}(u)_{e_j} \rightarrow (v)_{e_{j_1}}$ (this rule defines an arc $e_j \rightarrow e_{j_1}$).
 - ► $\{f\}(u, f)_{e_j} \to (v, g)_{e_j}$ (this rule defines an arc $e_j \to e_j$).
 - $f,g \in \Phi$, u, v are multisets over Σ and $1 \leq j, j_1 \leq m$.
- The environments are connected by arcs given by the system's rule set.
- $p_{\mathcal{R}_E}$ is a function that assigns a probability to each rule of the system.

Configurations

Remark

Configuration of $\Pi = (\Sigma, \Phi, (f_1, E_1), \dots, (f_m, E_m), \mathcal{R}_E, p_{\mathcal{R}_E})$ at an instant *t*:

- A tuple (x₁, u₁,..., x_m, u_m), where x_i ∈ Φ and u_i is a multiset over Σ (for 1 ≤ i ≤ m).
- The initial configuration of Π is $(f_1, E_1, \ldots, f_m, E_m)$.
- A rule of type $\{f\}(u)_{e_j} \xrightarrow{p_3} (v)_{e_{j_1}}$ is applicable to a configuration $C_t = (x_1, u_1, \ldots, x_m, u_m)$ of the system at a time instant t if $x_j = f$ and $u \subseteq u_j$.
 - The application of this rule removes the multiset u from the environment e_i and produces the multiset v in the environment e_i.

Configurations

Remark

- A rule of type $\{f\}(u, f)_{e_j} \xrightarrow{p_4} (v, g)_{e_j}$ is applicable to a configuration $C_t = (x_1, u_1, \dots, x_m, u_m)$ of the system at a time instant t if $x_j = f$ and $u \subseteq u_j$.
 - The application of this rule changes the flag of the environment e_j from f to g and replaces the multiset u with the multiset v in the environment e_j.
Dynamics of the population dynamics P systems

ad hoc algorithms

- the binomial block-based simulation algorithm (BBB) [Cardona et al., 2011],
- the direct non-deterministic distribution with probabilities algorithm (DNDP) [Colomer et al., 2010b],
- the direct distribution based on consistent blocks algorithm (DCBA) [Martínez-del-Amor et al., 2012]

- groups the rules with identical left-hand sides into blocks,
- the blocks are subjected to shuffling, and
- then the rules are executed in accordance with a multinomial random variate that is determined by the respective probabilities of the rules

DNDP

- employs a stochastic approach, iterating over the rules randomly
- for each rule, a binomial random variate is computed based on the associated probability
- the DNDP algorithm incorporates a second phase to ensure maximality
- in this stage, the remaining rules are executed sequentially to the maximum extent feasible

- divides the set of rules into consistent blocks
- distributes objects within these blocks proportionally
- \bullet the balance and consistency of the system \Longrightarrow accurate simulation of population dynamics

Stochastic vs probabilistic approach

- stochastic strategy for micro-level systems
 - stochastic multienvironment P systems = multicompartmental P systems
- probabilistic approach for macro-level modeling
 - probabilistic multienvironment P systems = population dynamics P systems

Applications

- modelling protein signalling pathways [Cheruku et al., 2007] ,
- gene expression control [Romero-Campero and Pérez-Jiménez, 2008b],
- molecular mechanisms of quorum sensing [Romero-Campero and Pérez-Jiménez, 2008a],
- spatial modelling of dynamical phenomena in ecology [Cardona et al., 2011], [Cardona et al., 2010], [Cardona et al., 2009], [Colomer et al., 2010], [Colomer et al., 2013], [Colomer et al., 2011], [Colomer et al., 2014], [Colomer et al., 2010a], [Duan et al., 2020], [Duan et al., 2023], [Martínez-del-Amor et al., 2012], [Rong et al., 2023], and
- simulating different epidemiological scenarios [Colomer et al., 2021].

Ecological modelling

- Identify the components of an ecosystem and the relationships/interactions among them.
- Explore how ecosystems respond to perturbations/disturbances.
- Focus on populations within ecosystems, the roles of species, how they shape their ecosystems.
- The decline of one population from an ecosystem affects the other players.
- The addition of a population (e.g., an invasive species or translocated herds) affects an ecosystem.
- Understand the interactions of biotic and abiotic factors within an ecosystem, the resistance and resilience of an ecosystem.

The modelling framework

- inherent modular, flexible, and extensible nature of membrane systems ⇒ the seamless integration of new components into the model (without major modifications)
- $\bullet\,$ modularity $\Longrightarrow\,$ reuse of existing structures and rules $\Longrightarrow\,$ more sophisticated model
- inclusion of an unlimited number of parameters and interactions \implies the dynamics and structural properties of complex systems

Our approach

- formulation of a computational models focussing on applications to ecology
- examining the diverse and complex interconnections and interaction patterns between entities and their respective environments
- challenge: extracting information pertaining to the interconnections and integrating this into the computational model
- designing the membrane structure of the multienvironment P system
 - the initial multisets symbolizing the entities and the rules abstracting the processes occurring within the system under study

- extending the various probabilistic strategies
- $\bullet\,$ enforcing the constraints on the model \Longrightarrow exploration of different scenarios

Our approach

- translation of the multienvironment P system into P-Lingua [Díaz-Pernil et al., 2009], [Martínez-del-Amor et al., 2020]
 - ► a standardized language for defining various types of P systems
- validation of the model against experimental data through the application of MeCoSim [MeCoSim], [Pérez-Hurtado et al., 2010]
 - ► a software toolkit for simulating biological phenomena in P systems

Search and selection of invasive / non-indigenous (NIS) species

- development of computational models for invasive species
- management tools for control and prevention

Detailed study of the species to be analyzed

- Task 1: General analysis of the invasion situation and selection of the species to be modelled.
- Task 2: Study of the main invasive species established in the specified habitat.
- Task 3: Study of species that may represent potential introduction threats in neighbouring areas.



Selection criteria



- invasive alien species
- unmanaged: lack of control measures in place for the management of their spread or impact
- real risk, well-demonstrated detrimental effects on ecosystem functions and services
- populations



• e.g., basin of the Guadalquivir river



Spain river basins, source: Wikipedia

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• e.g., inland wetlands



La Breña II in Córdoba, source: ABC de Córdoba



- occurrence data
- monitoring data
- species data
 - biology
 - ecology
 - life strategies

Applications in the field of ecology Selection







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Tier list



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Applications in the field of ecology Comprehensive inventory of NIS

NeoBiota 89: 17–44 (2023) doi: 10.3897/neobiota.89.105994 https://neobiota.pensoft.net

RESEARCH ARTICLE



A multi-taxa assessment of aquatic non-indigenous species introduced into Iberian freshwater and transitional waters

Jose M. Zamora-Marín^{1,2}, Ana Ruiz-Navarro^{1,3}, Francisco J. Oficialdegui^{1,4}, Pedro M. Anastácio⁵, Rafael Miranda⁶, Pablo García-Murillo⁷, Fernando Cobo⁸, Filipe Ribeiro⁹, Belinda Gallardo¹⁰, Emili García-Berthou¹¹, Dani Boix¹¹, Leopoldo Medina¹², Felipe Morcillo¹³, Javier Oscoz⁶, Antonio Guillén¹,

[Zamora-Marín et al., 2023]

Databases

- the European Alien Species Information Network (EASIN; http://easin.jrc.ec.europa.eu),
- CABI's Invasive Species Compendium (CABI-ISC; http://www.cabi.org/isc/),
- the Global Invasive Species Database (GISD; www.iucngisd.org),
- the EXOCAT database (http://exocatdb.creaf.cat/base_dades/#),
- the AquaNIS database (http://www.corpi.ku.lt/databases/aquanis/) and
- the Global Biodiversity Information Facility database (GBIF; http://www.gbif.org/)

Candidates

- Procambarus clarkii ("Red swamp crayfish", "Cangrejo rojo")
- Callinectes sapidus ("Atlantic blue crab", "Cangrejo azul")
- Azolla filiculoides ("Water fern", "Helecho de agua")
- Psittacula krameri ("Kramer parrot", "Cotorras de Kramer")



Procambarus clarkii, source: iagua

Procambarus clarkii

- native to Mexico, USA
- introduced everywhere (excl. Oceania, Antarctica)
- entered Europe via Seville
- good invader
 - generalist and resistant
 - transforms the landscape
 - aphanomycosis (genus of water molds)
 - batrachochytrium dendrobatidis (a fungal pathogen)
- there is monitoring, lots of people to ask
- 2020 thesis of bioaccumulation in P. clarkii (in the University of Seville itself)
- seems well studied



The World of Crayfish [lon et al., 2024]

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Callinectes sapidus, source: Las Provincias

Callinectes sapidus

- origin: Gulf of Mexico
- successful introduction only in Iberian Peninsula and Chile.
- invasive
 - generalist diet
 - tolerates a variety of conditions
 - carrier of parasites
- contemplated by the Seville Provincial Council
- present in La Algaba, Gillena, Salteras
- well-described and studied (since Darwin)



Azolla filiculoides, source: ABC Natural

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Azolla filiculoides

- native to the American tropics / introduced in many places
- good invader
 - forms a mat, denies light and causes anoxia
 - floats, drifts
 - reproduces by fragmentation
- CHG (Confederación Hidrográfica del Guadalquivir) has control programs
- there are detailed studies, people from the University of Seville

EL SPAÑOL



Una cotorra de Kramer ataca a un nóctulo gigante en el parque de María Luisa de Sevilla. Dallos Hernández-Brito

MEDIO AMBIENTE / ESPECIES INVASORAS

El parque de María Luisa de Sevilla presencia uno de los peores exterminios ecológicos del mundo

La población más importante del murciélago más grande de Europa está siendo erradicada por las cotorras invasoras.

15 mayo, 2018 - 01:53



Psittacula krameri, source: El Español

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