
An Improved Membrane Algorithm for Solving Time-Frequency Atom Decomposition

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Summary. To decrease the computational complexity and improve the search capability of quantum-inspired evolutionary algorithm based on P systems (QEPS), a real-observation QEPS (RQEPS) was proposed. RQEPS is a hybrid algorithm combining the framework and evolution rules of P systems with active membranes and real-observation quantum-inspired evolutionary algorithm (QEA). The RQEPS involves a dynamic structure including membrane fusion and division. The membrane fusion is helpful to enhance the information communication among individuals and the membrane division is beneficial to reduce the computational complexity. An NP complete problem, the time-frequency atom decomposition of noised radar emitter signals is employed to test the effectiveness and practical capabilities of the RQEPS. The experimental results show that RQEPS is superior to QEPS, the greedy algorithm and binary-observation QEA in terms of search capability and computational complexity.

1 Introduction

In 1998, Gheorghe Păun proposed membrane computing (P systems) [15][16]. A P system, employing various features to specify the structure and functionality of the living cells, is a membrane structure with objects in its membranes, with specified evolution rules like transformation/communication, merging and dividing membranes [15]. Until now, using the advantages of the new distributed parallel computing model and evolutionary algorithms (EAs), the combination technique of them, membrane algorithm, is applied to solve various complex problems. In [13] and [14], a membrane algorithm with a nested membrane structure was introduced to solve the travelling salesman problem as well as the min storage problem [10]. In

[7]-[9], a hybrid algorithm combining a P system with a conventional genetic algorithm (CGA) was proposed to solve single-objective and multi-objective numerical optimization problems. In [20], a hybrid distributed EA with membrane systems was presented to solve some continuous optimization problems. In [22], a membrane algorithm combining one level membrane structure with binary-observation quantum-inspired evolutionary algorithms (bQEA), called a QEA based on P systems (QEPS), was proposed to solve knapsack problems, and the experimental results show that QEPS performs better than its counterpart bQEA. But there are some drawbacks such as discretization error and Hamming cliff [6][24], when bQEA is used to solve numerical optimization problems. In [24], a real-observation QEA (RQEA) was proposed for numerical optimization problems to overcome the disadvantages of bQEA.

By combining RQEA with P systems with active membranes, this paper proposes an improved membrane algorithm, called a real-observation QEPS (RQEPS), to reduce the computational complexity and improve the search capability of QEPS [22][11]. In RQEPS, the real-observation rules are employed to connect quantum-inspired bit (Q-bit) representation and real-valued variables in each elementary membrane. And then all the elementary membranes are merged into one and all individuals in elementary membranes enter the merged membrane, where a copy of the best individual is sent out to the skin membrane. The recombination is operated on all individuals in the merged membrane to exchange the information among individuals. To demonstrate the effectiveness and applicability of the introduced method, experiments are carried out on the time-frequency atom decomposition (TFAD) of noised radar emitter signals to extend the application of the membrane algorithm. The experimental results show that RQEPS performs better than the greedy algorithm (GrA) [12], bQEA [6] and QEPS [22][11].

The TFAD is an approach that decomposes any signal into a linear combination of waveforms selected from a redundant dictionary of time-frequency atoms, which localized well both in time and frequency [12]. Differing from Fourier and Wavelet transforms, the information in TFAD is not diluted across the whole basis. Unlike Wigner and Cohen class distributions, the energy distribution obtained by TFAD does not include interference terms [12]. Hence, TFAD has become an important analysis technique in signal processing and harmonic analysis [12][17] [5]. One of the most successful methods for signal representations in over-complete dictionaries to solve this problem is the greedy algorithm (GrA) [12], but the extremely high computational load greatly blocks its practical applications. In [18][3][19][2], conventional genetic algorithms (CGAs) were introduced into TFAD to reduce the computational cost. However, due to slow convergence and premature convergence, it is difficult for CGAs to guide individuals toward better solutions in the search space. This paper uses a novel algorithm combining the framework of P systems with RQEA to reduce the computational load and improve the signal representation in the TFAD.

The remainder of this paper is organized as follows. Section 2 describes the TFAD and the pseudocode algorithm of EAs-based TFAD. Section 3 presents

the detailed algorithm for RQEPS. Section 4 discusses the number of elementary membranes, and conducts extensively comparative experiments on noised radar emitter signals. Finally, conclusions are drawn in Section 5.

2 Time-Frequency Atom Decomposition

The TFAD is an approach to select satisfactory time-frequency atoms $g_\gamma(t)_{\gamma \in \Gamma}$ from a redundant time-frequency atom dictionary $D = (g_\gamma(t))$ to decompose a signal into a linear combination of waveforms [12]. Let f be the original signal, $f \in H$, where H is a Hilbert space. When the signal f is decomposed up to the order $item$, f_{item} can be represented as

$$f_{item} = \sum_{n=0}^{item} \langle R^n f, g_{\gamma_n} \rangle g_{\gamma_n} + R^{item+1} f, \quad (1)$$

where g_{γ_n} satisfies

$$|\langle R^n f, g_{\gamma_n} \rangle| = \sup_{\gamma \in \Gamma} |\langle R^n f, g_\gamma \rangle|, \quad (2)$$

where $\Gamma = R^+ \times R^2$ is a set of indexes γ , and $R^{n+1} f$ is the residual signal

$$R^{n+1} f = R^n f - \langle R^n f, g_{\gamma_n} \rangle g_{\gamma_n}. \quad (3)$$

According to the conclusion [12]: $\lim_{item \rightarrow \infty} \|R^{item+1} f\| = 0$, the signal f_{item} can be represented as

$$f_{item} = \sum_{n=0}^{item} \langle R^n f, g_{\gamma_n} \rangle g_{\gamma_n}. \quad (4)$$

The problem of selecting a series of atoms to optimally approximate a signal in a redundant time-frequency atom dictionary is NP-hard [1]. One of the most successful methods to solve this problem is the greedy algorithm (GrA) [12]. GrA used a greedy strategy, in which the time-frequency atoms were selected one by one from an over-complete dictionary to best match the structure of signals [12][21]. However, as usual, the time-frequency dictionary is very large, so it is almost impossible for GrA to conduct the full search and represent the signals within a finite time, which seriously limits the practical application of TFAD. By the way, TFAD is a NP-hard problem. To decrease the computational efforts of TFAD, EAs were introduced into TFAD to search the suboptimal time-frequency atom from redundant time-frequency atom dictionaries [21]. The pseudocode algorithm for EAs-based TFAD is shown in Fig. 1. In this paper, an improved membrane algorithm, RQEPS is introduced into TFAD to decrease the computational complexity and improve the search capability, which will be presented in the next section.

```

Begin
  Initialization of TFAD; % Initial iteration  $item=1$ ;
  While (not termination condition) do
    Set parameters of time-frequency atom ;
    Search the suboptimal time-frequency atom in  $D$ 
    using EAs (RQEPS);
    Compute  $|\langle R^{item} f, g_{\gamma_{item}} \rangle g_{\gamma_{item}}|$ ;
     $R^{item} f \leftarrow (R^{item} f - \langle R^{item} f, g_{\gamma_{item}} \rangle g_{\gamma_{item}})$ ;
     $item = item + 1$ ;
  End while
End begin

```

Fig. 1. Pseudocode algorithm for EAs-based TFAD

3 An Improved Membrane Algorithm

The structure of an improved membrane algorithm, RQEPS is shown in Fig. 2, where the elementary membranes $1, 2, \dots, m$, embedded in the skin membrane 0, contain multisets of objects and evolution rules. In the computing process, all elementary membranes may be merged into one m_{in} for information communication and the merged membrane m_{in} may be divided into the same number of elementary membranes $1, 2, \dots, m$. The pseudocode algorithm of RQEPS is presented in Fig. 3 and the detailed description is as follows.

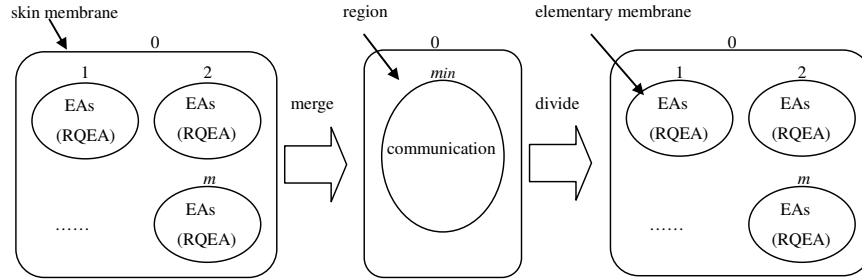


Fig. 2. The structure of RQEPS

(i) The membrane structure $[0[1]_1, [2]_2, \dots, [m]_m]_0$ is considered, in which the skin membrane S_0 contains m elementary membranes. The initial multisets:

$$\begin{aligned}
 S_0 &= \lambda, \\
 S_1 &= p_1 p_2 \cdots p_{n_1}, \quad n_1 \leq pop, \\
 S_2 &= p_{n_1+1} p_{n_1+2} \cdots p_{n_2}, \quad n_1 + n_2 \leq pop, \\
 &\dots \\
 S_m &= p_{n_{(m-1)}+1} p_{n_{(m-1)}+2} \cdots p_{n_m}, \quad n_1 + n_2 + \cdots + n_m \leq pop,
 \end{aligned}$$

```

Begin
(i)   Initializing the membrane structure; % gen=0;
      While (not termination condition) do
(ii)   Performing RQEA in all elementary membranes;
(iii)  Merging all elementary membranes into one and
        performing communication rules;
(iv)   Dividing the merged membrane;
        gen=gen+1;
      End while
End begin

```

Fig. 3. Pseudocode algorithm for RQEPS

where pop is the dimension of the population, and p_i , $1 \leq i \leq pop$, is a Q-bit individual of length n , which is represented as

$$\mathbf{p}_i^t = \begin{bmatrix} \alpha_{i1}|\alpha_{i2}| \cdots |\alpha_{in}| \\ \beta_{i1}|\beta_{i2}| \cdots |\beta_{in}| \end{bmatrix}, \quad (5)$$

where α_{ij} , β_{ij} are random numbers ranged from 0 to 1, and $|\alpha_{ij}|^2 + |\beta_{ij}|^2 = 1$, ($i = 1, 2, \dots, pop$, $j = 1, 2, \dots, n$).

(ii) The RQEA is performed in all elementary membranes. The pseudocode algorithm for RQEA is shown in Fig. 4, and the detailed description is as follows.

```

a) Set the iterations for each elementary membranes;
      For  $i=1:m$  do
           $t=0$ ;
b)   Generate  $R(t)$  by observing  $P(t)$ ;
c)   Evaluate  $R(t)$  and store the best solution among  $R(t)$ ;
      While (not termination condition) do
           $t=t+1$ ;
d)   Update  $P(t)$  using Q-gates;
e)   Make  $R(t)$  by observing the states of  $P(t)$ ;
f)   Evaluate  $R(t)$  and store the best solution among  $R(t)$ ;
      End while
End for

```

Fig. 4. Pseudocode algorithm for RQEA

a) The evolutionary generation t_i for RQEA in the i th elementary membrane is set to a uniformly random integer.

b) The states $R(t)$ in $P(t)$ are observed, where $R(t) = \{a_1^t, a_2^t, \dots, a_n^t\}$, and a_i^t ($i = 1, 2, \dots, n$) is an observed state of an individual p_i^t ($i = 1, 2, \dots, n$). a_i^t is a

real number of length n , that is $a_i^t = b_1 b_2 \cdots b_n$, where b_j^t ($j = 1, 2, \dots, n$) is a real number between 0 and 1. The observed states $R(t)$ are generated in probabilistic way. For instance, as for the probability amplitude $[\alpha, \beta]$ of a Q-bit, a random number r in the range $[0, 1]$ is generated. If $r < 0.5$, the corresponding observed value is set to $|\alpha|^2$, otherwise, the value is set to $|\beta|^2$.

c) Each individual is evaluated to give a measure of its fitness, and the best individual is stored. The fitness is evaluated to adapt the specific problem. In this paper, the fitness function is chosen as $|\langle R^{item}, g_{\gamma_{item}} \rangle g_{\gamma_{item}}|$, shown in Fig. 1.

d) In this step, the Q-bit individuals in $P(t)$ are updated by using quantum-inspired gates (Q-gates). A Q-gate is given by

$$G = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}, \quad (6)$$

where θ is the Q-gate rotation angle, and is defined as $\theta = k \cdot f(\alpha, \beta)$, where the value of k is chosen as [23]

$$k = 0.1\pi e^{-t/t_i}, \quad (7)$$

and $f(\alpha, \beta)$ are shown in Table 1.

The steps e) and f) are similar to steps b) and c), respectively.

Table 1. Look-up table of function $f(\alpha, \beta)$ [24], where sign is a symbolic function

$\xi_1 > 0$	$\xi_2 > 0$	$f(\alpha, \beta)$	
		$ \xi_1 \geq \xi_2 $	$ \xi_1 < \xi_2 $
True	True	+1	-1
True	False	$sign(\alpha_1, \alpha_2)$	
False	True	$-sign(\alpha_1, \alpha_2)$	
False	False	$sign(\alpha_1, \alpha_2)$	$-sign(\alpha_1, \alpha_2)$
$\xi_1, \xi_2 = 0$ or $\pi/2$		± 1	

(iii) Except for the skin membrane, all elementary membranes are merged into one m_{in} , and consequently the objects of all elementary membranes enter the membrane m_{in} . Subsequently, the communication rules are performed in the membrane m_{in} , that is, a copy of the best element P_{best} , selected in merged membrane, is sent out to the skin membrane. The recombination operation conducted in the merged membrane is used to exchange the information among individuals, which is shown in Fig. 5, where p_i and p_j are any arbitrary two individuals in m_{in} and p'_i and p'_j are the recombined individuals.

(iv) The membrane m_{in} is divided into the same structure with the m elementary membranes. In the process of division, the copies of objects $p_1 p_2 \cdots p_{n_1}$

$$\left\{ \begin{array}{l} p_i \begin{bmatrix} \alpha_{i1} | \alpha_{i2} | \dots | \alpha_{ih} | \dots | \alpha_{in} \\ \beta_{i1} | \beta_{i2} | \dots | \beta_{ih} | \dots | \beta_{in} \end{bmatrix} \\ p_j \begin{bmatrix} \alpha_{j1} | \alpha_{j2} | \dots | \alpha_{jh} | \dots | \alpha_{jn} \\ \beta_{j1} | \beta_{j2} | \dots | \beta_{jh} | \dots | \beta_{jn} \end{bmatrix} \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} p'_i \begin{bmatrix} \alpha_{i1} | \alpha_{i2} | \dots | \beta_{jh} | \dots | \alpha_{in} \\ \beta_{i1} | \beta_{i2} | \dots | \alpha_{jh} | \dots | \beta_{in} \end{bmatrix} \\ p'_j \begin{bmatrix} \alpha_{j1} | \alpha_{j2} | \dots | \beta_{ih} | \dots | \alpha_{jn} \\ \beta_{j1} | \beta_{j2} | \dots | \alpha_{ih} | \dots | \beta_{jn} \end{bmatrix} \end{array} \right\}$$

Fig. 5. The recombination operation

are sent into the membrane S_1 ; the copies of objects $p_{n_1+1}p_{n_1+2} \dots p_{n_2}$ are sent into the membrane S_2 and the rest may be deduced by analogy. Finally, the copy of P_{best} is sent from the skin membrane to each compartment to determine the Q-gate rotation angle at the next generation.

RQEPS is an improved algorithm of the QEPS [22]. The differences between these two approaches are as follows.

(a) They use different observation rules: binary-observation rules in QEPS [22] vs. real-observation rules in RQEPS. In RQEPS, a quantum-inspired state, corresponding to an optimization variable, observed by a real-observation rule is a real-valued number. But an optimization variable in QEPS needs several quantum-inspired states, which correspond with a string of binary bits in the binary-observation process. Without encoding and decoding processes, the real-observation rule is more suitable for solving numerical optimization problems.

(b) Preliminary use of membrane fusion and division is considered in RQEPS.

(c) Recombination operations are employed in merged membrane to exchange the information among individuals.

4 Experimental Results

In this section, how to choose the number m of elementary membranes will be first discussed by using a linear frequency-modulated radar emitter signal with 10 dB signal-to-noise rate (SNR), shown in Fig. 6. And then the comparative experiments are carried out on the signal to demonstrate the effectiveness and applicability of the introduced method.

4.1 Parameter Setting

In this subsection, experiments on the noised signal are carried out to investigate the effects of the number m of elementary membranes on the performance of RQEPS for TFAD. Experimental environment is chosen as: the maximal number of iterations *item* is set to 30 as the termination condition of TFAD. The time-frequency atom uses Gabor function

$$g_\gamma(t) = \frac{1}{\sqrt{s}} g\left(\frac{t-u}{s}\right) \cos(vt + w), \quad (8)$$

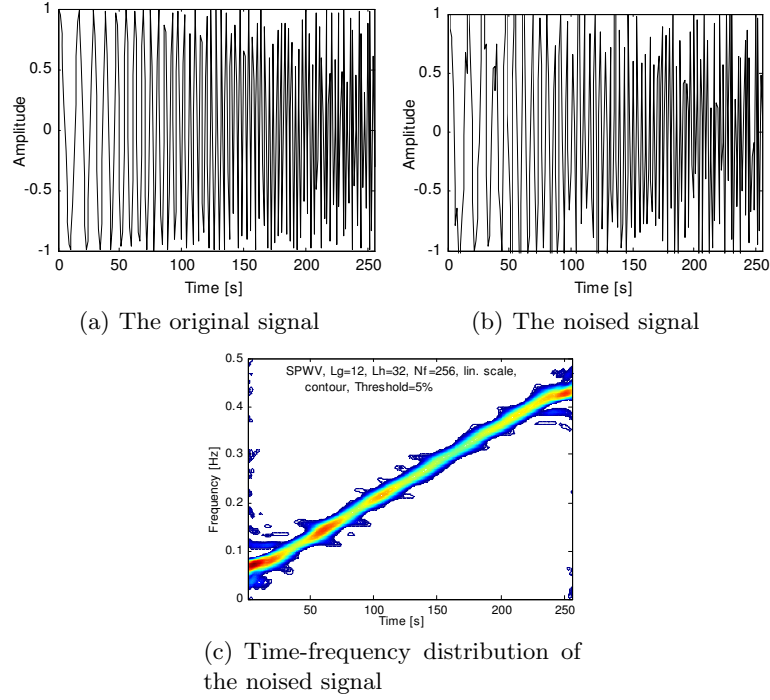


Fig. 6. A radar emitter signal

where the index $\gamma = (s, u, v, w)$ is a set of parameters and s, u, v, w are scale, translation, frequency and phase, respectively. They are discretized as follows: $\gamma = (a^j, pa^j \Delta u, ka^{-j} \Delta \xi, i \Delta w)$, $a = 2, \Delta u = 1/2, \Delta \xi = \pi, \Delta w = \pi/6, 0 < j < \log_2 N$, $0 \leq p \leq N2^{-j+1}, 0 \leq k < 2^{j+1}, 0 \leq i \leq 12$, where N is the length of the signal f [12].

In RQEPS, the population size pop is set to 10. The parameter m varies from 2 to 10. According to the investigation of the effect of the parameter t_i ($i = 1, 2, \dots, m$) on the QEPS performances in [22], the RQEA's iteration t_i is set to a uniformly random integer ranged from 1 to 9. The number n of a Q-bit individual and the maximal evolutionary generation gen are set to 4 and 40, respectively. These experiments are carried out on the computer with 1.5 GHz CPU, 768 MB EMS memory and 80GB hard disk using the software MATLAB 7.1. The experimental results over 30 runs as the number of elementary membranes are shown in Fig. 7, which illustrates that the elapsed time, the mean best and the variance best of the correlation ratio C_r between the original signal f and the restored signal f_{res} . The correlation ratio C_r of f and f_{res} is defined as [25]

$$C_r = \frac{\langle f, f_{res} \rangle}{\|f\| \cdot \|f_{res}\|}, \quad (9)$$

The experimental results in Fig. 7(a) and 7(b) show that the mean and the variance of the best correlation ratio C_r show a broad range of variability with respect to the number of different elementary membranes, but the best results are obtained in two cases including 2 elementary membranes. As shown in Fig.7(c), the elapsed time has a steady increase with the number of the elementary membranes. Thus, to obtain the balance between the elapsed time and the correlation ratio, the number of elementary membranes could be assigned as 2.

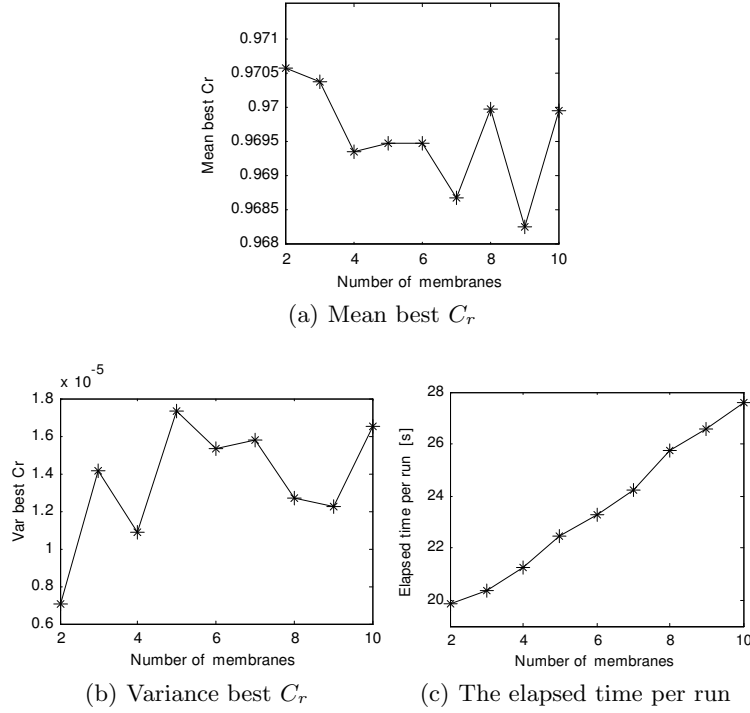


Fig. 7. Experimental results with different elementary membranes

4.2 Comparative Experiments

To verify the validity of RQEPS, the noised signal above is used to conduct the experiments with the same computer, in which bQEA [6], GrA [12] and QEPS [22][11] are brought into comparisons with RQEPS.

In bQEA, population size pop , the number n of binary bits and the maximal evolutionary generation g are set to 10, 40 and 200, respectively. In QEPS, according to [11], the number m of elementary membranes is set to 9; the number n of binary bits is set to 40. In RQEPS, according to the experiments discussed in the

above subsection, the number m of elementary membranes is set to 2; the number n of a Q-bit individual is set to 4. In both RQEPS and QEPS, the parameter t_i ($i = 1, 2, \dots, m$) is set to a uniformly random integer ranged from 1 to 9; the population size pop and the maximal evolutionary generation gen are set to 10 and 40, respectively. In all algorithms, the maximal number of iterations item is set to 30 as the termination condition of TFAD. Experimental results are shown in Fig. 8 to Fig. 11, Table 2, Table 3 and Table 4.

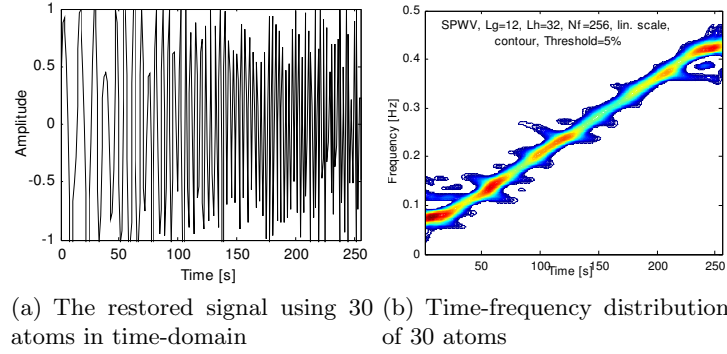


Fig. 8. Experimental results obtained by bQEA

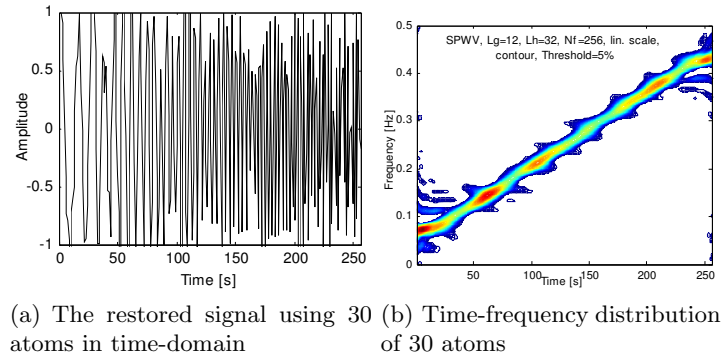


Fig. 9. Experimental results obtained by GrA

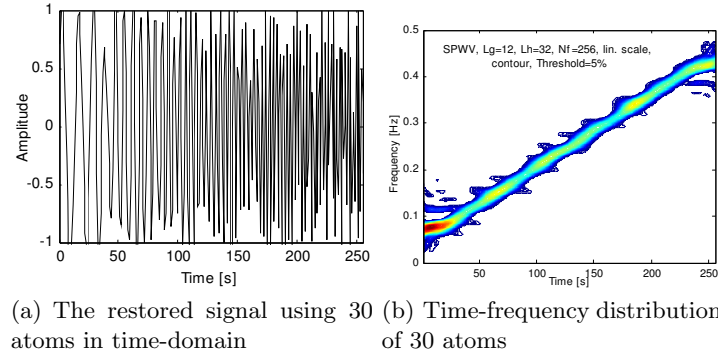


Fig. 10. Experimental results obtained by QEPS

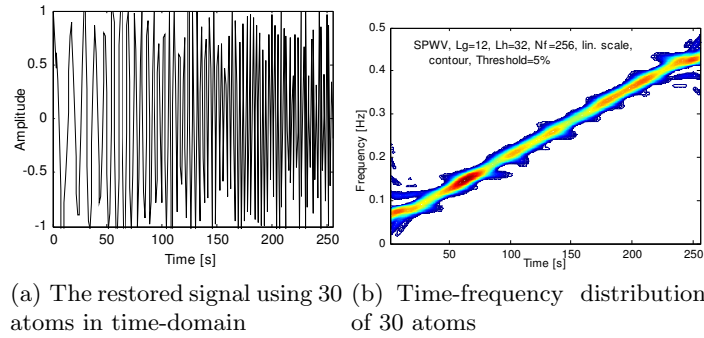


Fig. 11. Experimental results obtained by RQEPS

Table 2 lists the parameters of the 30 Gabor atoms. Fig. 8 to Fig. 11 show the restored signals using the 30 decomposed time-frequency atoms and their time-frequency distributions of the 30 time-frequency atoms which are obtained by bQEA, GrA, QEPS and RQEPS, respectively. As shown in Fig.6 and Fig. 8 to Fig.11, it can be seen that the time-frequency distribution obtained by RQEPS is nearly identical with that of the original radar emitter signals, and the correlation ratio is the highest which reaches 0.9801, while the correlation ratio obtained by GrA is only 0.9668, which illustrates that RQEPS is more suitable for decomposing a signal into time-frequency atoms than bQEA, GrA and QEPS, in terms of search capability.

The experimental results over 30 runs are shown in Table 3 and Table 4. From Table 3, it can be seen that RQEPS gains the mean of the best correlation ratio C_r 0.9706, which is better than 0.9670, 0.9668 and 0.9505 obtained by QEPS, GrA and bQEA, respectively. Moreover, the computing time of RQEPS is 36.4061, 2.2441, and 2.1766 times as small as that of GrA, QEPS and bQEA. If the experiments are conducted in a parallel-distributed way on several machines, the computing time could be greatly reduced.

Table 2. Parameters of 30 atoms of a noised LFM radar emitter signal

	1	2	3	4	5	6	7	8	9	10
s	19.63	22.71	26.83	43.94	29.27	28.97	28.41	33.70	33.25	12.32
u	99.43	136.29	209.41	55.67	177.86	22.28	237.06	76.69	157.50	6.46
v	1.31	1.63	3.89	0.85	2.05	0.57	2.68	1.00	1.85	5.67
w	3.71	3.51	4.64	5.08	4.15	3.06	4.37	2.17	2.45	1.56
	11	12	13	14	15	16	17	18	19	20
s	33.87	34.51	90.38	1.73	31.82	10.29	10.29	31.73	22.65	12.32
u	120.19	193.4	45.05	0.19	49.52	80.16	224.93	251.23	100.67	196.40
v	4.85	2.20	5.36	4.40	0.68	1.14	3.77	3.42	1.50	3.95
w	3.91	2.87	4.09	3.99	0.75	3.46	3.21	0.05	3.63	4.24
	21	22	23	24	25	26	27	28	29	30
s	13.42	25.77	8.12	15.26	25.39	12.92	9.10	17.11	6.33	28.53
u	181.09	37.99	5.28	100.34	122.04	45.84	81.22	152.62	132.19	243.11
v	2.07	0.59	0.02	4.23	1.84	2.41	5.78	1.38	5.85	0.52
w	1.61	3.21	0.58	1.45	2.27	2.91	1.87	4.08	2.72	3.92

Table 3. Performance comparisons of bQEA, GrA, QEPS and RQEPS

	Correlation ratio C_r		Computing time per run (Second)
	Mean	Var	
bQEA	0.9505	7.2387e-5	43.25
GrA	0.9668	1.1476e-31	723.39
QEPS	0.9670	1.2400e-5	44.59
RQEPS	0.9706	7.0583e-6	19.87

Table 4. Results of parametric statistical test t -test

Control Algorithm	bQEA	GrA	QEPS
RQEPS	8.0113e-18	1.1684e-10	4.7336e-05

In table 4, a parametric statistical analysis t -test is applied to analyse whether there is a significant difference over one optimization problem between two algorithms [4]. We employ a 95% confidence Student t -test. The t -test results in Table 4 are far smaller than the level of significance 0.05, which implies that RQEPS really outperforms the QEPS, GrA and bQEA by introducing the active membranes with merge and division operations, real-observation and recombination operations.

5 Conclusions

This paper proposes an improved membrane algorithm (RQEPS), by combining the framework and evolution rules of P systems with RQEA. RQEPS is characterized by active membranes with fusion and division membranes to strengthen the information communication among individuals and decrease the computational complexity, respectively, the evolutionary rules in RQEA and transformation/communication like-rules in P systems to evolve the system. The TFAD of noised radar emitter signals is considered as an application example to test the effectiveness and practicality of the introduced method. Experimental results show that RQEPS performs better than QEPS, GrA and bQEA, in terms of search capability and convergent speed.

The possible interplay between evolutionary algorithms and membrane computing represents a challenging and promising research topic. This paper introduces RQEA into P systems to solve time-frequency atom decomposition. However, how to select evolutionary algorithms within elementary membranes and communication rules in the merged membrane to solve different complex problems, in order to obtain more efficient methods, is an ongoing and challenging issue.

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