
P Systems with Reactive Membranes

Artiom Alhazov¹, Rudolf Freund², Sergiu Ivanov³, David Orellana-Martín^{4,5},
Antonio Ramírez-de-Arellano^{4,5}, José Antonio Rodríguez Gallego⁶

¹Vladimir Andrunachievici Institute of Mathematics and Computer Science,
The State University of Moldova, Academiei 5, Chişinău, MD-2028, Moldova
`artiom@math.md`

²Faculty of Informatics, TU Wien
Favoritenstraße 9–11, 1040 Wien, Austria
`rudi@emcc.at`

³IBISC Laboratory, Université Paris-Saclay, Univ Évry
91020, Évry-Courcouronnes, France
E-mail : `sergiu.ivanov@univ-evry.fr`

⁴Research Group on Natural Computing,
Department of Computer Science and Artificial Intelligence,
Universidad de Sevilla
Avda. Reina Mercedes s/n, 41012 Sevilla, Spain
E-mail: `{dorellana,aramirezdearellano}@us.es`

⁵SCORE Laboratory, I3US, Universidad de Sevilla
Avda. Reina Mercedes s/n, 41012 Sevilla, Spain

⁶Departamento de Construcciones Arquitectónicas I,
Universidad de Sevilla
Avda. Reina Mercedes 2, 41012 Sevilla, Spain
E-mail: `jrodriguez14@us.es`

Summary. Membranes are one of the key concepts in P systems and membrane computing, and a lot of research focuses on their properties and possible extensions: membrane division, membrane dissolution, mobile membranes, etc. In this work, we explore the possibility of using membranes for thinking about the emergence of milieu separations at the origins of life. We propose a new variant of P systems with reactive membranes, in which every symbol is initially surrounded by an elementary membrane, and in which membranes can non-deterministically merge and split, leading to the formation of bigger and more complicated membranes. We show that such non-deterministic splitting and merging does not seem to radically affect the computational power: P systems with reactive membranes and non-cooperative rules generate at least all semilinear languages, and cooperative rules allow for simulating partially blind register machines. We briefly discuss using P systems with reactive membranes for illustrating the emergence of autocatalytic cycles, but actual constructions are left for future work.

Keywords: origins of life, P systems, self-assembly, space and topology.

1 Introduction

Membrane computing is a multiset rewriting-based theoretical construct for natural computing, originally introduced by Gh. Păun in [23], and extensively studied ever since. The structure of a membrane system—or a P system—mimics that of a living cell: it is a hierarchical family of nested membranes, each carrying a multiset of abstract objects and multiset rewriting rules. The objects can be seen as formal representations of chemical species, and the rewriting rules capture the biochemical interactions these species may have.

Beyond the obvious abstraction arrow between biochemical species and formal objects, membrane computing parallels biological systems in another interesting way. In biology, centralization of functions is quite frequent (e.g., central nervous systems, specialized organs, etc.), but not fundamental. Only as a first example, simple organisms carry out many activities in a decentralized way, weakly orchestrated by interference between related processes. Take unicellular organisms: a computer scientist may be tempted to consider the genetic material as the program for the whole cell, but it is now known (e.g. [10]) that the relationship between the genotype and the phenotype—its manifestation—is very far from the clear program–execution duality imbuing computer science. As an abstraction of hierarchically structured biochemistry, P systems inherit this weakly centralized way of functioning, which makes them a good candidate for supporting the thought process about some grand laws of biology.

In this paper, we lay the groundwork for using P systems as a tool for thinking about some aspects of the emergence of life. The particular question we focus on is the emergence of milieu separations, which played an essential role as they allowed to isolate and protect relevant processes from the environment [11]. Since P systems already include membranes as first-class citizens, we will use them as a framework for thinking about the emergence of complex regions from simpler ones.

The approach we take here is to posit that every copy of a symbol a is endowed with some *elementary space*—a membrane which initially only contains the multiset a . Two such symbols can bond by merging their membranes, thereby yielding a more complex membrane containing 2 symbols. Such membranes can further merge, yielding bigger and bigger regions. Dually, membranes containing multiple symbols can split into a pair of simpler membranes, with the content of the original larger membrane distributed across its children. This is in fact membrane separation (e.g. [7, 21, 22]).

Measuring the complexity of a membrane by the number of symbols it contains is simultaneously simple and appropriate: cooperative evolution rules are allowed, so more symbols means more applicable rules and therefore more interactions. In the setup we establish in this paper, all membranes share the same common set of evolution rules. The rules can naturally be seen as defining a chemistry, while membrane merging and splitting can on the other hand be seen as some lower-level ground laws governing who may interact with whom, i.e. the topology of the interactions. The resulting abstract structures featuring merging and splitting

membranes are therefore systems in which objects interact based on the non-deterministic variations in their neighborhoods. We call such structures P systems with *reactive membranes*.

Before using P systems with reactive membranes as a formal tool, a number of important details have to be sorted out. In particular, we show that the definition of membrane splitting and merging turns out rather nontrivial. Choosing when to recover and how to interpret the result also impacts the form of the computations of a P system with reactive membranes, and also what kind of results one can expect. Finally, this P system variant as informally introduced above and defined in Section 3 is very basic and may be extended in many ways, as we briefly show in Section 5.

Note that we do not pretend to faithfully model in any way the processes which happened at the origins of life. Rather, we acknowledge the exceptional complexity of these processes, as well as the impossibility to experimentally verify any of the related hypotheses (e.g., [17]). The intended role of P systems with reactive membranes is to serve as a formal vehicle for an otherwise abstract thought process, to help verify the latter in a basic way, and to help the researcher to deal with complex questions. This approach is similar in spirit to the works [26, 27], in which sign Boolean networks are used with a similar purpose.

P systems with reactive membranes are naturally part of the lineage of P systems with active membranes, and feature similarities with other variants in this family. Among closely related variants are P systems with mobile membranes, in which membranes are allowed to move across the membrane structure, and thereby change their immediate neighbors [8, 9, 20]. Another variant are P systems with vesicles of multisets, in which multisets are contained in vesicles, which are contained in membranes, implying that entire multisets of symbols can travel between different membranes, thereby activating different sets of rules [5, 15]. A key specificity of P systems with reactive membranes setting them apart from the other variants is that membrane splitting and merging is global, compulsory, and independent of the contents of the membranes or of the rules. This feature introduces a basic form of space, through which the entities travel and in which they interact in their immediate neighborhood. On the other hand, such compulsory splitting and merging modulates the computational power in interesting ways.

This paper is structured as follows. In Section 2 we recall some basic concepts from formal languages and P systems. In Section 3 we introduce P systems with reactive membranes, and define the precise semantics of splitting and merging of membranes. In Section 4 we present some first results concerning the computational power of P systems with reactive membranes, with non-cooperative and cooperative rules. In Section 5 we give some examples of possible extensions to the new variant. Finally, in Section 6, we discuss the potential of reactive membranes for illustrating some processes which happened at the origins of life, as well as some aspects of their computational power.

2 Preliminaries

For an alphabet V , a finite non-empty set of abstract symbols, the free monoid generated by V under the operation of concatenation, i.e., the set containing all possible strings over V , is denoted by V^* . The empty string is denoted by λ , and $V^* \setminus \{\lambda\}$ is denoted by V^+ .

For two natural numbers $a, b \in \mathbb{N}$, $a \leq b$, we use the notation $[a..b]$ to refer to the interval of natural numbers between a and b , both included: $[a..b] = \{a, a + 1, \dots, b\}$.

Given a finite set A , a multiset over A is a function $w : A \rightarrow \mathbb{N}$, assigning the number of times an element of A appears in w . The infinite set of all multisets over A is denoted by A° . The family of finite sets of finite multisets over A is denoted by $\mathcal{P}_{fin}(A^\circ)$.

To spell out a multiset w , we will generally write any string containing exactly the same symbols with the same multiplicities. For example, the strings aab , aba , ba^2 will be used to refer to the same multiset w with the property $w(a) = 2$, $w(b) = 1$, and $w(c) = 0$ for all $c \in A \setminus \{a, b\}$. We denote the empty multiset by λ , i.e. $\forall a \in A : \lambda(a) = 0$, and its string representation is λ , the empty string.

Given two multisets w_1 and w_2 over A , their multiset union $w_1 \cup w_2$ is defined as $(w_1 \cup w_2)(a) = w_1(a) + w_2(a)$, for all $a \in A$. As their multiset intersection $w_1 \cap w_2$ we define $(w_1 \cap w_2)(a) = \min\{w_1(a), w_2(a)\}$. A restriction of the multiset $w : A \rightarrow \mathbb{N}$ to the subset $B \subseteq A$ is the multiset $w|_B : A \rightarrow \mathbb{N}$ with the property that $w|_B(a) = w(a)$ if $a \in B$ and $w|_B(a) = 0$ otherwise.

The family of regular, context-free, and recursively enumerable string languages is denoted by $\mathcal{L}(REG)$, $\mathcal{L}(CF)$, and $\mathcal{L}(RE)$, respectively. For a family of languages FL , the family of Parikh images of languages in FL is denoted by $PsFL$. As $Ps\mathcal{L}(REG) = Ps\mathcal{L}(CF)$, in the area of multiset rewriting $\mathcal{L}(CF)$ plays no role at all, and in the area of membrane computing we often only get characterizations of $Ps\mathcal{L}(REG)$ and $Ps\mathcal{L}(RE)$.

For further notions and results in formal language theory we refer to textbooks like [12] and [25].

In the rest of this section, we briefly recall P systems and the related concepts. For more extensive overviews, we refer the reader to [18, 24].

A (*transition*) *P system* is a construct

$$\Pi = (O, T, \mu, w_1, \dots, w_n, R_1, \dots, R_n, \delta, h_i, h_o) \text{ where}$$

- O is the alphabet of *objects*,
- $T \subseteq O$ is the alphabet of *terminal objects*,
- μ is the *membrane structure* injectively labelled by the numbers from $\{1, \dots, n\}$ and usually given by a sequence of correctly nested brackets,
- w_i are the multisets giving the *initial contents* of each membrane i , $1 \leq i \leq n$,
- R_i is the finite *set of rules* associated with membrane i , $1 \leq i \leq n$,
- δ is the *derivation mode*, and

- h_i and h_o are the labels of the *input membrane* and the *output membrane*, respectively; $1 \leq h_i \leq n, 1 \leq h_o \leq n$).

Taking into account the well-known flattening process, which means that computations in a P system with an arbitrary (static) membrane structure can be simulated in a P system with only one membrane, e.g., see [14], often only *simple P systems* are considered, i.e., with the simplest membrane structure of only one membrane region, and then we write:

$$\Pi = (O, T, w_1, R_1, \delta)$$

Quite often, the rules associated with membranes are multiset rewriting rules (or special cases of such rules). Multiset rewriting rules have the form $u \rightarrow v$, with $u \in O^\circ \setminus \{\lambda\}$ and $v \in O^\circ$, where O° is the set of multisets over O , and $\lambda(a) = 0$, for all $a \in O$. If $|u| = 1$, the rule $u \rightarrow v$ is called *non-cooperative*, otherwise it is called *cooperative*. In *communication P systems*, rules are additionally allowed to send symbols to the neighbouring membranes. In this case, for rules in R_i , $v \in (O \times Tar_i)^\circ$, where Tar_i contains the symbols *out* (corresponding to sending the symbol to the parent membrane), *here* (indicating that the symbol should be kept in membrane i), and *in_j* (indicating that the symbol should be sent into the child membrane j of membrane i). When writing out the multisets over $O \times Tar_i$, the indication *here* is often omitted.

In P systems, rules are often applied in the maximally parallel way: in one derivation step, only a non-extendable multiset of rules can be applied. The rules are not allowed to consume the same instance of a symbol twice, which creates competition for objects and may lead to non-deterministic choice between the maximal collections of rules applicable in one step. The *maximally parallel derivation mode* is generally denoted by the symbol *max*. Other derivation modes include the *sequential derivation mode sequ* in which exactly one rule is applied in every step, the *set maximally parallel derivation mode smax* only allowing multisets of rules in which every rule has multiplicity 1, as well as the *asynchronous derivation mode asyn* under which no restriction is imposed on the applied multiset of rules. We refer to the works [3, 4, 6, 16] for an in-depth discussion of the matter.

A *computation* of a P system is traditionally considered to be a sequence of configurations it can successively visit, stopping at the halting configuration. A *halting* configuration is a configuration in which no rule can be applied any more, in any membrane. The *result of a computation* in a P system Π as defined above is the contents of the output membrane h_o projected over the terminal alphabet T .

We will use the notations $N(\Pi)$ and $Ps(\Pi)$ to respectively refer to the number language and the language of multisets generated by Π . The notation $OP_n(\delta, \tau)$ will refer to the family of P systems with at most n membranes, operating under the derivation mode δ and relying on the rules of type τ , where $\tau = coo$ if cooperative rules are allowed and $\tau = ncoo$ if all rules are non-cooperative. Finally, we use the notations $NOP_n(\delta, \tau)$ and $PsOP_n(\delta, \tau)$ to refer to the family of number languages and multiset languages, respectively, generated by the P systems in the family $OP_n(\delta, \tau)$.

Example 1. Figure 1 shows the graphical representation of the P system formally given by

$$\begin{aligned} \Pi &= (\{a, b, c, d\}, \{a, d\}, [1[2]_2]_1, R_1, R_2, 1, 1), \\ R_2 &= \{a \rightarrow aa, b \rightarrow b(c, out)\}, \\ R_1 &= \emptyset. \end{aligned}$$

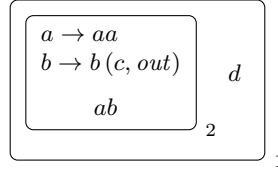


Fig. 1. An example of a simple P system.

In the maximally parallel mode, the inner membrane 2 of Π will apply as many instances of the rules as possible, thereby doubling the number of a , and ejecting a copy of c into the surrounding (skin) membrane in each step. The symbol d in the skin membrane is not used. Therefore, after k steps of evolution, membrane 2 will contain the multiset $a^{2^k}b$ and membrane 1 the multiset $c^k d$. Since all rules are always applicable in Π , this P system never halts. \square

3 Reactive Membranes

A P system with reactive membranes is the following construct:

$$\Pi = (O, T, W_0, R, \delta) \text{ where}$$

- O is the alphabet of *objects*,
- $T \subseteq O$ is the alphabet of *terminal objects*,
- $W_0 \in \mathcal{P}_{fin}(O^\circ)$ is the (finite) *initial set of multisets* over O ,
- $R \subseteq O^\circ \times O^\circ$ is the set of *evolution rules*, and
- δ is the *derivation mode*.

We will require that at least one of the sides of all rules in R be non-empty, i.e.

$$\forall u \rightarrow v \in R : u \neq \lambda \vee v \neq \lambda.$$

We immediately stress two major features of this definition. On the one hand, we do not include any membrane structure. Indeed, as W_0 hints, we simply use individual multisets to represent the contents of the individual membranes, without explicitly representing the membranes themselves. Incidentally, this means that membranes do not nest in this model. On the other hand, the evolution of all symbols in all multisets is governed by the same common set of rules R .

A configuration of Π is any set of multisets over O . Similarly to networked models of computing like networks of evolutionary processors (e.g. [19]) or tissue P systems with vesicles of multisets [5], a computation step in P systems with reactive membranes consists of two stages:

1. splitting and merging,
2. evolution.

Informally, the splitting and merging stage implements the non-deterministic evolution of the membranes—individual multisets under this definition—as described in the introduction: any two multisets may merge, and any multiset may split in two. The evolution stage consists in applying the evolution rules in R to every multiset of the configuration, according to the mode δ . In the following paragraphs we give a formal description of both stages, applied to a configuration $W_i \in \mathcal{P}_{fin}(O^\circ)$.

Splitting and merging stage

1. Non-deterministically partition W_i into 3 subsets:

$$W_i = M_i \cup S_i \cup I_i$$

such that $|M_i|$ is even, and the sets S_i , M_i , and I_i are mutually disjoint, i.e., $S_i \cap M_i = S_i \cap I_i = M_i \cap I_i = \emptyset$. The multisets in M_i will be merged pairwise, the multisets in S_i will be split, and the multisets in I_i will remain intact.

2. Partition M_i into a set of disjoint pairs. Non-deterministically pick a bijection $\varphi : [1..|M_i|] \rightarrow M_i$ and construct the following set:

$$\hat{M}_i = \{(\varphi(2k-1), \varphi(2k)) \mid 1 \leq k \leq |M_i|/2\}.$$

Then define $M'_i = \{w_1 \cup w_2 \mid (w_1, w_2) \in \hat{M}_i\}$.

3. Define $\text{split}(w)$ to be the set of all possible ways to split the multiset w into two multisets:

$$\text{split}(w) = \{(w_1, w_2) \mid w_1 \cup w_2 = w, w_1, w_2 \in O^\circ\}.$$

Define the set of all possible ways of splitting the multisets in S_i :

$$\hat{S}_i = \prod_{w \in S_i} \text{split}(w).$$

Non-deterministically pick $S'_i \in \hat{S}_i$.

4. Compute the new intermediate configuration as

$$W'_i = M'_i \cup \text{flatten}(S'_i) \cup I_i,$$

where $\text{flatten}(S'_i) = \{w_1, w_2 \mid (w_1, w_2) \in S'_i\}$.

In the above presentation we describe merging before splitting, but the order of the two substeps does not matter, since they occur on disjoint sets M_i and S_i . Furthermore, we stress that multiple intermediate configurations W'_i may be obtained from the same configuration W_i .

Evolution stage

The evolution stage is defined in the conventional way by applying the rules in R to every multiset in W'_i individually, according to the derivation mode δ :

$$W_i = \{w' \in W'_i \mid w \xrightarrow{\delta, R} w'\},$$

where w' is a multiset derived from w by applying the rules in R under the mode δ .

A configuration W is *halting* if no rules are applicable in the evolution stage, for any intermediate configuration W' which can be obtained from W in the splitting and merging stage. An *n-step halting computation* of a P system with reactive membranes Π is a finite sequence of configurations $(W_i)_{0 \leq i \leq n}$ such that W_{i+1} is obtained from W_i by the computation step described above, and W_n is a halting configuration.

As the *result of a computation* in a P system with reactive membranes Π as defined above we take all the terminal objects appearing in the membranes present in a halting configuration W_n :

$$\left(\bigcup_{w \in W_n} w \right) \Big|_T = \bigcup_{w \in W_n} w|_T.$$

To conclude the introduction of P systems with reactive membranes, we again stress that the splitting and merging of multisets (or membranes) is non-deterministic, imposed in every computation step, and independent of the features of the configuration or of the rules in R . More concretely, the rules in R cannot directly influence which symbols will appear next to which after the splitting and merging stage.

Example 2. Consider the following P system with reactive membranes:

$$\begin{aligned} \Pi &= (O, T, W_0, R, \max), \text{ where} \\ O &= \{a, b, c, d, e, f\}, \\ T &= \{d, f\}, \\ W_0 &= \{a, b, c\}, \\ R &= \{ab \rightarrow d, abc \rightarrow f, a \rightarrow e\}. \end{aligned}$$

For the first step of the computation, Π may decide to not split or merge any multisets ($M_0 = S_0 = \emptyset$, $I_0 = W_0$), meaning that the evolution rules will be applied directly to singleton multisets a , b , and c . While no rules are applicable to b or c individually, the rule $a \rightarrow e$ will have to be applied to a , yielding the next configuration $W_1 = \{b, c, e\}$. We can immediately conclude that the rules $ab \rightarrow d$ and $abc \rightarrow f$ will never be applicable any more later in this computation, as there is no way to reintroduce a . In sum, this halting computation yields the result A .

Suppose now that Π decides to merge the multisets a and b in the first step, yielding the intermediate configuration $W'_0 = \{ab, c\}$. In this case a non-deterministic choice will appear in the evolution stage between applying the rule $ab \rightarrow d$ or $a \rightarrow e$ (both singleton sets of rules are non-extendable). As a consequence, the following two possibilities exist for the second configuration: $\{d, c\}$ and $\{eb, c\}$. In sum, these halting computations yield the results d and Λ , respectively.

Finally, note that the rule $abc \rightarrow f$ will never be applicable with $W_0 = \{a, b, c\}$, since putting a , b , and c together in one membrane requires at least two mergers, and a will necessarily be consumed by $a \rightarrow e$ or $ab \rightarrow d$ along the way. On the other hand, if we put together a , b and c in one multiset from the start, or even if we put ab together and c apart, the rule $abc \rightarrow f$ will have a chance to be applied. In particular, in the case in which the initial configuration is $\{ab, c\}$ it suffices to consider the branch of the computation along which Π decides to merge the two multisets in the first splitting and merging stage. In sum, with the initial sets $\{ab, c\}$ and $\{a, b, c\}$ we can get the results Λ , d , and f . \square

As indicated by the example discussed above, it makes a difference in how many multisets the initial multiset of objects is divided. Thus, we will use the notation $Re_nOP(\delta, \tau)$ to refer to the family of P systems with reactive membranes starting with n initial multisets, running under the mode δ and using rules of type $\tau \in \{coo, ncoo\}$, as well as the notations $NRe_nOP(\delta, \tau)$ and $PsRe_nOP(\delta, \tau)$ to refer to the family of number languages and multiset languages, respectively, generated by the P systems with reactive membranes from $Re_nOP(\delta, \tau)$.

Whereas on the one hand the previous example shows the effect of having more than one initial membrane, prohibiting the application of some evolution rules, the next example shows that the halting condition can be fulfilled due to the fact that symbols are distributed over several membranes, although some rule could be applied if all symbols on its left-hand side could be put into the same membrane by a merge operation. As merging can only combine the contents of two membranes, we can already get the situation that a rule with three symbols in its left-hand side cannot be applied any more.

Example 3. Consider the following P system with reactive membranes:

$$\begin{aligned} \Pi &= (O, T, W_0, R, max), \text{ where} \\ O &= \{a_i, a'_i, a''_i \mid 1 \leq i \leq 3\} \cup \{f\}, \\ T &= \{a''_1, a''_2, a''_3, f\}, \\ W_0 &= \{a_1 a_2 a_3\}, \\ R &= \{a_i \rightarrow a'_i, a'_i \rightarrow a''_i, a''_1 a''_2 a''_3 \rightarrow f\}. \end{aligned}$$

If in the first two steps of the computation, Π decides to not split or merge any multisets, from $W_0 = \{a_1 a_2 a_3\}$ with applying the rules $\{a_i \rightarrow a'_i \mid 1 \leq i \leq 3\}$, after the first evolution step we obtain $W_1 = \{a'_1 a'_2 a'_3\}$, and by then applying the rules $\{a'_i \rightarrow a''_i \mid 1 \leq i \leq 3\}$, after the second evolution step we obtain $W_2 = \{a''_1 a''_2 a''_3\}$. Keeping $\{a''_1 a''_2 a''_3\}$ in the same membrane then allows for applying

the rule $a_1''a_2''a_3'' \rightarrow f$, thus obtaining the terminal result $W_3 = \{f\}$, as W_3 is a halting configuration.

Yet with two splits, but still applying the rules $\{a_i \rightarrow a_i' \mid 1 \leq i \leq 3\}$ in the first evolution step and the rules $\{a_i' \rightarrow a_i'' \mid 1 \leq i \leq 3\}$ in the second evolution step, we get a two-step halting computation

$$\{a_1a_2a_3\} \Longrightarrow \{a_1', a_2'a_3'\} \Longrightarrow \{a_1'', a_2'', a_3''\}$$

yielding the terminal result $a_1''a_2''a_3''$.

We also mention that with having $T = \{f\}$ only, this halting computation yields the result Λ . \square

4 Computational Power: First Results

In this section, we list some first results regarding the computational power of P systems with reactive membranes. We start by remarking that the halting condition can be checked in an easier way when the system only includes non-cooperative rules.

Remark 1. When using only non-cooperative rules, the halting condition for a configuration W can be checked without considering all possible splits and mergers and then the non-applicability of the rules in all membranes; instead it suffices to check the non-applicability of the rules to the $\text{flatten}(W)$, i.e., to the union of multisets in all the membranes of W .

The following result even holds for non-cooperative rules and cooperative rules.

Lemma 1. *For every $Re_1OP(\delta, \tau)$ system there exist an equivalent $Re_nOP(\delta, \tau)$ system, for every $n > 1$.*

Proof. Given a P system with reactive membranes using rules of type τ

$$\Pi' = (O, T, \{w\}, R, \delta),$$

an equivalent P system with reactive membranes using rules of type τ with n initial membranes is

$$\Pi' = (O, T, \{w, w_2 = \Lambda, \dots, w_n = \Lambda\}, R, \delta).$$

In an empty membrane Λ , no non-cooperative rules or cooperative rules are applicable. Moreover, merging a membrane X with Λ yields X again, so no additional applications of rules can happen.

Now we show that splitting and merging do not affect the (results of the) computations in a P system with reactive membranes at all, no matter which derivation mode is used, when only non-cooperative rules are used. Hence, we get a characterization of $Ps\mathcal{L}(REG)$:

Theorem 1. For any $\delta_1, \delta_2 \in \{asyn, seq, max, smax\}$ and $Y \in \{N, Ps\}$ as well as any $n \geq 1$,

$$YRe_nOP(\delta_1, ncoo) = YOP_1(\delta_2, ncoo) = Y\mathcal{L}(REG).$$

Proof. The equality $YOP_1(\delta_2, ncoo) = Y\mathcal{L}(REG)$ is folklore, e.g., see [24]. The main idea for proving this result is that the evolution of symbols by applying non-cooperative rules can be described by a derivation tree, but for the resulting terminal objects it is completely irrelevant when the symbols evolve.

A similar argument now can be used here to argue that for any $\delta_1 \in \{asyn, seq, max, smax\}$,

$$YRe_nOP(\delta_1, ncoo) = YOP_1(asyn, ncoo) = Y\mathcal{L}(REG).$$

(\Rightarrow) Given a P system with reactive membranes

$$\Pi' = (O, T, \{w\}, R, \delta_1),$$

we can easily define the equivalent simple P system

$$\Pi = (O, T, w, R, asyn).$$

Even distributing the contents of a single membrane over several membranes, even at the beginning with having several initial multisets, does not effect the applicability of the non-cooperative rules. Yet we have to mention that using the sequential derivation mode in several membranes yields a kind of parallelism like *smax*, but also this has no effect on the results of computations, especially as, according to Remark 1, halting only depends on the non-applicability of all rules to the symbols in all the multisets of the underlying configuration.

(\Leftarrow) Given a simple P system

$$\Pi = (O, T, w, R, asyn),$$

we can easily define the equivalent P system with reactive membranes

$$\Pi' = (O, T, \{w\}, R, asyn).$$

Any derivation of the 1-membrane transition P system Π operating under the the asynchronous derivation mode can be directly simulated by the P system with reactive membranes Π' which uses the same rules for the evolution stage, but then always chooses to not split or merge any membranes, i.e. M_i and S_i from the splitting and merging stage are always empty. As we are only using non-cooperative rules, the applicability of all the (multisets of) rules applied in Π is also guaranteed in Π' .

Finally we can apply Lemma 1 to get an equivalent P system with reactive membranes with n initial membranes.

In sum we see that P systems with reactive membranes behave as the corresponding transition P system when only non-cooperative rules are used. \square

Finally, P systems with reactive membranes working under the maximally parallel mode and using cooperative rules can simulate partially blind register machines. As a reminder, we mention that partially blind register machines (PBRM) have programs consisting of the following two types of instructions for incrementing and decrementing a register:

- $(p, \text{ADD}(r), q, s)$: in state p increment register r and jump to state q or state s ;
- $(p, \text{SUB}(r), q)$: in state p try to decrement register r ; if successful, jump to state q , otherwise abort the computation without producing a result.

Partially blind register machines feature a final zero check: the register machine only halts with producing a result if all non-output registers are empty when the machine reaches the halting instruction uniquely labeled by h .

We will refer to the set of multiset languages generated by partially blind register machines by $PsPBRM$.

Theorem 2. For any $\delta \in \{asyn, sequ, max, smax\}$,

$$PsPBRM \subseteq PsRe_1OP(\delta, coo).$$

Proof (Sketch). The main idea of the proof is that throughout the simulation of the partially blind register machine, the configurations of the P system with reactive membranes Π always contains exactly one instance of the symbol representing the label of the instruction to be carried out next. The contents of a register r is represented by the total number of symbols a_r in the configurations of Π .

The increment instruction $(p, \text{ADD}(r), q, s)$ can be simulated directly by the rules $p \rightarrow qa_r$ and $p \rightarrow sa_r$.

The decrement instruction $(p, \text{SUB}(r), q)$ can be simulated by the following two rules: $pa_r \rightarrow q$, $p \rightarrow p$. Moreover, for every register symbol a_r with r not being an output register, we add the unit rules $a_r \rightarrow a_r$.

Indeed, if p and a copy of a_r find themselves in the same membrane, then a successful decrement is simulated: the total number of copies of a_r in the system is reduced by one.

If there are no copies of a_r left in the system, then p only has the chance to be used with the unit rule $p \rightarrow p$; observe that in any derivation mode at least one rule has to be applied if the system is not halting, i.e., as long as there still is a rule which can be applied to some symbol. In this case, either $p \rightarrow p$ and/or some unit rule $a_r \rightarrow a_r$ can be applied in every future derivation step, hence, the computation will never halt.

If copies of a_r do appear in the system, but not in the membrane containing p , then p can use the unit rule $p \rightarrow p$, and in any derivation mode either only this rule and/or other unit rules $a_r \rightarrow a_r$ can be applied. If in some future step, p and a_r appear in the same membrane, possibly $pa_r \rightarrow q$ can be applied. Otherwise, again we obtain just non-halting computation branches.

However, there must exist another branch in which no splits and mergers have happened at all, i.e., p and a_r are together, and in which the simulation therefore will be able to proceed correctly. The same alternative holds if p and a_r share the same membrane, but the system non-deterministically would choose to only apply $p \rightarrow p$ rather than $pa_r \rightarrow p$.

As soon as the halting label h appears, we have to use the final rule $h \rightarrow \lambda$. The final zero check is simulated by the unit rules $a_r \rightarrow a_r$ for all non-output registers r , which keep the computation to go on forever if at least one such symbol a_r is still present. Observe that this argument does not depend on the distribution of the symbols in the membranes of a configuration.

In sum, we conclude that the P system with reactive membranes Π can simulate the computations of the given partially blind register machine correctly, but on the other hand cannot yield more results. \square

Finally, we remark that the construction we show here is non-deterministic, even if the simulated partially blind register machine is deterministic, i.e., all increment instructions are of the form $(p, \text{ADD}(r), q, q)$, which in a simpler way can be written as $(p, \text{ADD}(r), q)$.

5 Extensions

Given the motivation to use P systems with reactive membranes for thinking about the emergence of space and space separations in abiotic environments, and also the richness of the ecosystem of P systems variants, multiple extensions can be proposed.

A natural one to be considered would be *limiting the size* of individual membranes, as real membranes do not generally grow very big. Limitations on the number of symbols have already been considered in P systems [2], but combined with constant splitting and merging this ingredient may have a drastically different impact. It would be necessary to decide what happens when a membrane attains its maximal capacity. The approach in [2] is to prevent it to accept new symbols, but in the context of reactive membranes it may be appropriate to bias the splitting and merging stage of the computational step to force such full membranes to split. The contribution of such limitations to the computational power is yet unclear, but probably in some strong relation to the size of the left-hand sides of the evolution rules.

An extension in the spirit of generalized P systems [13] would be to *subject the rules to splitting and merging*. With such an extension, membranes would contain objects and rules, and splitting and merging would affect not only which symbols can interact, but also which rules will ensure their interaction.

Finally, splitting and merging could also be applied to rules: for example, a rule $u \rightarrow v$ could split into two rules $u \rightarrow \alpha$ and $\alpha \rightarrow v$, which could later merge back into $u \rightarrow v$. Similarly to splitting and merging of membranes, splitting and

merging of rules delays some interactions. Relevance to thinking about the origins of life and the computational power of this variant remain to be explored.

6 Conclusion and Perspectives

This paper is a first attempt at using P systems for thinking about the origins of life, and in particular about the emergence of individual compartments separated by membranes. We introduced P systems with reactive membranes, in which every symbol is conceptually surrounded by elementary membranes, which then can merge to form bigger membranes, or split. Mimicking biochemistry, the set of rules is common to all membranes—the differences in the processes in different membranes should come from the symbols. Cooperative rules are allowed, and probably even necessary to meaningfully implement distinctions between membranes.

It is still an open research direction to actually illustrate some processes believed to have happened during abiogenesis in P systems with reactive membranes. Perhaps the most promising would be to implement autocatalytic cycles (e.g. [11]). The next step would be to implement *self-replication*, as suggested by José M. Sempere in a discussion. Indeed, in P systems with reactive membranes the membrane structure emerges spontaneously, which makes them a promising candidate for implementing self-replication of something other than symbol objects.

A parallel research direction which we started to explore in this paper is the computational power of P systems with reactive membranes. We have shown here that splitting and merging does not affect the computational power of P systems with reactive membranes using non-cooperative rules—P systems with reactive membranes using non-cooperative rules have the same computational power as simple P systems provided we only start with one singleton multiset, no matter which derivation mode we use. Based on this result, we have shown that P systems with reactive membranes can characterize the family of Parikh sets of semilinear languages when using only non-cooperative rules in any derivation mode.

Finally, when cooperative rules are allowed, P systems with reactive membranes can generate all multiset languages generated by partially blind register machines.

Several questions still remain to be addressed, in particular: can splitting and merging *augment* the computational power? It would indeed be surprising, but it has already been shown that non-deterministic shuffling of rule right-hand sides allows for generating non-semilinear languages [1], meaning that random shuffling of symbol neighborhoods as described in this paper may boost the power of the variant in some specific cases.

A subtle aspect which we do not discuss in depth in this paper is the halting condition and the procedure for retrieving the result. There is an asymmetry between these two: halting occurs when no more evolution rules are applicable after all possible splits and mergers. On the other hand, getting out the result essentially happens by merging *all* membranes into a single one.

Since the computational results we give in this paper seem to depend directly on the halting condition and on the procedure for obtaining the result, it would

be relevant to explore how slight variations in these two affect the computational power of P systems with reactive membranes.

Finally, in Section 5 we have suggested several possible extensions of the new variant. A formal exploration of the computational power of such extensions would be quite relevant. Even more importantly, it would be very relevant to identify which extensions are more useful for using P systems with reactive membranes in thinking about the origins of life.

Acknowledgements

The authors would like to thank the Organizing Committee of the 19th Brainstorming Week on Membrane Computing¹ (BWMC 2023) for organizing this fruitful event. The authors would also like to thank José M. Sempere for multiple helpful suggestions about self-replication and emergence of space.

Artiom Alhazov acknowledges project 20.80009.5007.22 “Intelligent information systems for solving ill-structured problems, processing knowledge and big data” by the National Agency for Research and Development.

References

1. Artiom Alhazov, Rudolf Freund, and Sergiu Ivanov. P systems with randomized right-hand sides of rules. *Theor. Comput. Sci.*, 805:144–160, 2020.
2. Artiom Alhazov, Rudolf Freund, and Sergiu Ivanov. P systems with limited number of objects. *J. Membr. Comput.*, 3(1):1–9, 2021.
3. Artiom Alhazov, Rudolf Freund, Sergiu Ivanov, and Marion Oswald. Variants of derivation modes for which purely catalytic P systems are computationally complete. *Theor. Comput. Sci.*, 920:95–112, 2022.
4. Artiom Alhazov, Rudolf Freund, Sergiu Ivanov, and Sergey Verlan. Variants of derivation modes for which catalytic P systems with one catalyst are computationally complete. *J. Membr. Comput.*, 3(4):233–245, 2021.
5. Artiom Alhazov, Rudolf Freund, Sergiu Ivanov, and Sergey Verlan. Tissue P systems with vesicles of multisets. *Int. J. Found. Comput. Sci.*, 33(3&4):179–202, 2022.
6. Artiom Alhazov, Rudolf Freund, and Sergey Verlan. P systems working in maximal variants of the set derivation mode. In Alberto Leporati, Grzegorz Rozenberg, Arto Salomaa, and Claudio Zandron, editors, *Membrane Computing – 17th International Conference, CMC 2016, Milan, Italy, July 25-29, 2016, Revised Selected Papers*, volume 10105 of *Lecture Notes in Computer Science*, pages 83–102. Springer, 2016.
7. Artiom Alhazov and Tseren-Onolt Ishdorj. Membrane operations in P systems with active membranes. *Second Brainstorming Week on Membrane Computing*, pages 37–44, 2004.
8. Bogdan Aman and Gabriel Ciobanu. Simple, enhanced and mutual mobile membranes. *Trans. Comp. Sys. Biology*, 11:26–44, 2009.

¹ <http://www.gcn.us.es/19bwmc>

9. Luca Cardelli and Gheorghe Păun. An universality result for a (mem)brane calculus based on mate/drip operations. *Int. J. Found. Comput. Sci.*, 17(1):49–68, 2006.
10. Matthew Cobb. 60 years ago, Francis Crick changed the logic of biology. *PLOS Biology*, 15(9):1–8, 09 2017.
11. Bruce Damer and David Deamer. The hot spring hypothesis for an origin of life. *Astrobiology*, 20(4):429–452, 2020.
12. Jürgen Dassow and Gheorghe Păun. *Regulated Rewriting in Formal Language Theory*. Springer, 1989.
13. Rudolf Freund. Generalized P-systems. In Gabriel Ciobanu and Gheorghe Păun, editors, *Fundamentals of Computation Theory, 12th International Symposium, FCT '99, Iasi, Romania, August 30 - September 3, 1999, Proceedings*, volume 1684 of *Lecture Notes in Computer Science*, pages 281–292. Springer, 1999.
14. Rudolf Freund, Alberto Leporati, Giancarlo Mauri, Antonio E. Porreca, Sergey Verlan, and Claudio Zandron. Flattening in (tissue) P systems. In Artiom Alhazov, Svetlana Cojocaru, Marian Gheorghe, Yurii Rogozhin, Grzegorz Rozenberg, and Arto Salomaa, editors, *Membrane Computing*, volume 8340 of *Lecture Notes in Computer Science*, pages 173–188. Springer, 2014.
15. Rudolf Freund and Marion Oswald. Tissue P systems and (mem)brane systems with mate and drip operations working on strings. In Nadia Busi and Claudio Zandron, editors, *Proceedings of the First Workshop on Membrane Computing and Biologically Inspired Process Calculi, MeCBIC@ICALP 2006, Venice, Italy, July 9, 2006*, volume 171 (2) of *Electronic Notes in Theoretical Computer Science*, pages 105–115. Elsevier, 2006.
16. Rudolf Freund and Sergey Verlan. A formal framework for static (tissue) P systems. In George Eleftherakis, Petros Kefalas, Gheorghe Păun, Grzegorz Rozenberg, and Arto Salomaa, editors, *Membrane Computing, 8th International Workshop, WMC 2007, Thessaloniki, Greece, June 25-28, 2007. Revised Selected and Invited Papers*, volume 4860 of *Lecture Notes in Computer Science*, pages 271–284. Springer, 2007.
17. Nicolas Glade. *Le Vivant Rare, Faible et Amorphe - Évolution depuis les Origines jusqu'à la Vie telle qu'elle nous Apparait. (Rare, Weak and Amorphous Life - Evolution of Life from the Origins until Life as it Appears Nowadays)*. HAL Open Archive, 2022.
18. Bulletin of the International Membrane Computing Society (IMCS). <http://membranecomputing.net/IMCSBulletin/index.php>.
19. Sergiu Ivanov, Yurii Rogozhin, and Sergey Verlan. Small universal networks of evolutionary processors. *J. Autom. Lang. Comb.*, 19(1-4):133–144, 2014.
20. Shankara Narayanan Krishna and Gheorghe Păun. P systems with mobile membranes. *Nat. Comput.*, 4(3):255–274, 2005.
21. David Orellana-Martín, Luis Valencia-Cabrera, and Mario J. Pérez-Jiménez. P systems with evolutionary communication and separation rules. In Jérôme Durand-Lose and György Vaszil, editors, *Machines, Computations, and Universality – 9th International Conference, MCU 2022, Debrecen, Hungary, August 31 – September 2, 2022, Proceedings*, volume 13419 of *Lecture Notes in Computer Science*, pages 143–157. Springer, 2022.
22. Linqiang Pan and Tseren-Onolt Ishdorj. P systems with active membranes and separation rules. *J. Univers. Comput. Sci.*, 10(5):630–649, 2004.
23. Gheorghe Păun. Computing with membranes. *Journal of Computer and System Sciences*, 61(1):108–143, 2000.

24. Gheorghe Păun, Grzegorz Rozenberg, and Arto Salomaa, editors. *The Oxford Handbook of Membrane Computing*. Oxford University Press, 2010.
25. Grzegorz Rozenberg and Arto Salomaa, editors. *Handbook of Formal Languages*. Springer, 1997.
26. Rémi Segretain, Sergiu Ivanov, Laurent Trilling, and Nicolas Glade. A methodology for evaluating the extensibility of Boolean networks' structure and function. In Rosa M. Benito, Chantal Cherifi, Hocine Cherifi, Esteban Moro, Luis Mateus Rocha, and Marta Sales-Pardo, editors, *Complex Networks & Their Applications IX - Volume 2, Proceedings of the Ninth International Conference on Complex Networks and Their Applications, COMPLEX NETWORKS 2020, 1-3 December 2020, Madrid, Spain*, volume 944 of *Studies in Computational Intelligence*, pages 372–385. Springer, 2020.
27. Rémi Segretain, Laurent Trilling, Nicolas Glade, and Sergiu Ivanov. Who plays complex music? On the correlations between structural and behavioral complexity measures in sign Boolean networks. In *21st IEEE International Conference on Bioinformatics and Bioengineering, BIBE 2021, Kragujevac, Serbia, October 25-27, 2021*, pages 1–6. IEEE, 2021.