Minimal cooperation in polarizationless P systems with active membranes. Complexity aspects

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Some complexity classes beyond NP and co-NP

DP: class of "differences" of any two languages in NP (Papadimitriou and Yannakakis, 1984).

\[ \text{DP} = \{ L \mid \exists L_1, L_2 (L_1 \in \text{NP} \wedge L_2 \in \text{co-NP} \wedge L = L_1 \cap L_2) \}. \]

\[ \text{NP} \cup \text{co-NP} \subseteq \text{DP}. \]

SAT-UNSAT problem.

Remark: If \( X \) is an NP-complete problem such that \( X \in \text{PMC}_R \) (\( R \) is a class of recognizer membrane systems stable under product family), then \( \text{DP} \subseteq \text{PMC}_R \).

PP: the majority of possible solutions associated with each instance is yes (Gill, 1977).

\[ \text{DP} \subseteq \text{PP} \text{ and } \text{PH} \subseteq \text{P}^{\text{PP}}. \]

MAJORITY-SAT problem.

\( \#P \): counting problems associated with polynomially balanced polynomial-time decidable relations (Valiant, 1979).

\[ \text{PP} \preceq \#P \subseteq \text{PSPACE} \text{ and } \text{PH} \subseteq \text{P}^{\#P}. \]

\( \#\text{SAT} \) problem.
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Basic polarizationless P systems with active membranes

\[ \Pi = (\Gamma, H, \mu, M_1, \ldots, M_q, R, i_{\text{out}}) \] of degree \( q \geq 1 \):

\( \Gamma \) is a finite alphabet whose elements are called objects;

\( H \) is a finite alphabet such that \( H \cap \Gamma = \emptyset \) whose elements are called labels;

\( \mu \) is a labelled rooted tree consisting of \( q \) nodes injectively labeled by elements of \( H \);

\( M_1, \ldots, M_q \) are multisets over \( \Gamma \);

\( R \) is a finite set of rules, of the following forms:

- \((a_0)[a \rightarrow u]h\) (object evolution rules).
- \((b_0)a[b \rightarrow h]h\) (send–in communication rules).
- \((c_0)a[h \rightarrow b][h]h\) (send–out communication rules).
- \((d_0)a[h \rightarrow b][h]h\) (dissolution rules).

\( i_{\text{out}} \in H \cup \{\text{env}\} \) (if \( i_{\text{out}} \in H \) then \( i_{\text{out}} \) is the label of a leaf of \( \mu \)).
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The class \( \mathcal{NAM}^0 \).
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The class \( \mathcal{NAM}^0 \).

It is well known that \( \text{PMC}_{\mathcal{NAM}^0} = \mathbb{P} \).
Division rules and separation rules

Mechanisms to produce an exponential workspace in linear time:

• Cell division: basic process in the cell life cycle producing two or more cells from one cell (its contents is replicated between the new membranes).

⋆ Division rules for elementary membranes: $[a]h ightarrow [b]h[c]h$

⋆ Division rules for non–elementary membranes: $[[a]h]h_0 ightarrow [[b]h]h_0[[c]h]h_0$

The class $DAM_0(±e, ±c, ±d, ±n)$.

• Membrane fission: process by which a biological membrane is split into two new ones (its contents is distributed between the new membranes).

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Efficiency of $DAM^0(+e, +c, +d, +n)$
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**Theorem:** Subset-Sum $\in \text{PMC}_{\mathcal{DAM}^0(+e, +c, +d, +n)}$ (2005).

**Corollary:** DP $\subseteq \text{PMC}_{\mathcal{DAM}^0(+e, +c, +d, +n)}$. 
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Are necessary division rules for non-elementary membranes?
Păun’s conjecture

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At the beginning of 2005, Gh. Păun wrote (problem F from [1]):

My favorite question (related to complexity aspects in P systems with active membranes and with electrical charges) is that about the number of polarizations. Can the polarizations be completely avoided? The feeling is that this is not possible – and such a result would be rather sound: passing from no polarization to two polarizations amounts to passing from non–efficiency to efficiency.

This so–called Păun’s conjecture can be formally formulated as follows:

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This so–called Păun’s conjecture can be formally formulated as follows:

\[ \text{PMC}_{\text{DAM}^0(+e,+c,+d,-n)} = \text{P} \]

An affirmative answer: the ability to create an exponential amount of workspace in polynomial time, is not enough in order to solve computationally hard problems efficiently.

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An **affirmative answer**: the ability to create an exponential amount of workspace in polynomial time, is not enough in order to solve computationally hard problems efficiently.

A **negative answer**: provide a borderline between tractability and intractability (assuming that \(\mathcal{P} \neq \mathcal{NP}\)).

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Partial solutions to Păun’s conjecture
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**Theorem:** \( \text{PMC}_{DAM^0(\text{+e,+c,}\text{-d,+n})} = \text{P} \) (2005).

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What syntactical ingredients are enough to solve $\textbf{NP}$-complete problems in an efficient way, by using the frameworks $\mathcal{DAM}^0(-d,-n)$ or $\mathcal{SAM}^0(-d,-n)$?
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**Dissolution:** An apparently innocent rule.
Minimal cooperation in object evolution rules

\[ u \rightarrow v \] for \( h \in H \) and \( u, v \in M_f(\Gamma) \) such that \( 1 \leq |u| \leq 2 \)

Bounded minimal cooperation (bmc):

\[ u \rightarrow v \] for \( h \in H \) and \( u, v \in M_f(\Gamma) \) such that \( 1 \leq |v| \leq |u| \leq 2 \)

Primary minimal cooperation (pmc):

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Minimal cooperation and minimal production (mcmp):

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\[ mc = \Rightarrow pmc = \Rightarrow bmc = \Rightarrow mcmp \]

The class \( DAM_0(\alpha, +\epsilon, -\delta, \pm n) \), where \( \alpha \in \{mc, pmc, bmc, mcmp\} \).
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The class \( \mathcal{DAM}^0(\alpha, +c, -d, \pm n) \), where \( \alpha \in \{mc, pmc, bmc, mcmp\} \).
Bounded minimal cooperation

Object evolution rules: \([a \rightarrow b]_h ; [a b \rightarrow c]_h ; [a b \rightarrow c d]_h\)
Bounded minimal cooperation

Object evolution rules: \( [a \rightarrow b]_h ; [a b \rightarrow c]_h ; [a b \rightarrow c d]_h \)

**Theorem:** \( \text{SAT} \in \text{PMC}_{\mathcal{DAM}}^0(\mathcal{bmc},+c,-d,-n) \cdot \)

**Corollary:** \( \text{DP} \subseteq \text{PMC}_{\mathcal{DAM}}^0(\mathcal{bmc},+c,-d,-n) \cdot \)
Bounded minimal cooperation

Object evolution rules: $\left[ a \rightarrow b \right]_h; \left[ a b \rightarrow c \right]_h; \left[ a b \rightarrow c d \right]_h$

Theorem: $\text{SAT} \in \text{PMC}_{\text{DAM}^0}(\text{bmc}, +c, -d, -n)$.  

Corollary: $\text{DP} \subseteq \text{PMC}_{\text{DAM}^0}(\text{bmc}, +c, -d, -n)$.  

Theorem: $\text{PMC}_{\text{SAM}^0}(\text{bmc}, +c, -d, +n) = \text{P}$.  

Bounded minimal cooperation

Object evolution rules: \([ a \rightarrow b ]_h ; [ a b \rightarrow c ]_h ; [ a b \rightarrow c d ]_h\)

**Theorem:** \(\text{SAT} \in \text{PMC}_{DAM^0(bmc,+c,−d,−n)}\).

**Corollary:** \(\text{DP} \subseteq \text{PMC}_{DAM^0(bmc,+c,−d,−n)}\).

**Theorem:** \(\text{PMC}_{SAM^0(bmc,+c,−d,+n)} = P\).

* New frontier of the efficiency in the framework \(AM^0(bmc,+c,−d,−n)\): separation versus division.

* New frontier of the efficiency in the framework \(DAM^0(∗,+c,−d,−n)\): non-cooperation in object evolution rules versus \(bmc\) in object evolution rules.
Primary minimal cooperation

Object evolution rules: 
- $[a \rightarrow b]_h$
- $[a \rightarrow b c]_h$
- $[a b \rightarrow c]_h$
- $[a b \rightarrow c d]_h$

Theorem: SAT $\in$ PMC DAM 0 (pmc, +c, −d, −n) ∩ PMC SAM 0 (pmc, +c, −d, −n).

Corollary: DP $\subseteq$ PMC DAM 0 (pmc, +c, −d, −n) ∩ PMC SAM 0 (pmc, +c, −d, −n).

⋆ New frontier of the efficiency in the framework SAM 0 (∗, +c, −d, −n):
- bmc versus pmc
- non-cooperation in object evolution rules versus pmc in object evolution rules.
Primary minimal cooperation

Object evolution rules: \[ a \rightarrow b \] \( h \); \[ a \rightarrow bc \] \( h \); \[ ab \rightarrow c \] \( h \); \[ ab \rightarrow cd \] \( h \)

Theorem: \( SAT \in PMC_{DAM^0(pm, +c, \d, \n)} \cap PMC_{SAM^0(pm, +c, \d, \n)} \).

Corollary: \( DP \subseteq PMC_{DAM^0(pm, +c, \d, \n)} \cap PMC_{SAM^0(pm, +c, \d, \n)} \).
Primary minimal cooperation

Object evolution rules: \([ a \rightarrow b ]_h \); \([ a \rightarrow b c ]_h \); \([ a b \rightarrow c ]_h \); \([ a b \rightarrow c d ]_h \)

Theorem: \( \text{SAT} \in \text{PMC}_{\mathcal{D}A\mathcal{M}^0(\text{pmc},+c,-d,-n)} \cap \text{PMC}_{\mathcal{S}A\mathcal{M}^0(\text{pmc},+c,-d,-n)} \).

Corollary: \( \text{DP} \subseteq \text{PMC}_{\mathcal{D}A\mathcal{M}^0(\text{pmc},+c,-d,-n)} \cap \text{PMC}_{\mathcal{S}A\mathcal{M}^0(\text{pmc},+c,-d,-n)} \).

★ New frontier of the efficiency in the framework \( \mathcal{S}A\mathcal{M}^0(\text{mc},+c,-d,-n) \): \textit{bmc} versus \textit{pmc}.

★ New frontier of the efficiency in the framework \( \mathcal{S}A\mathcal{M}^0(\star,+c,-d,-n) \): \textit{non-cooperation} in object evolution rules versus \textit{pmc} in object evolution rules.
Minimal cooperation and minimal production

Object evolution rules: $[a \rightarrow b]_h ; [a \ b \rightarrow \ c]_h$

Theorem: $\text{SAT} \in \text{PMC}_0(\text{mcmp}, +c, -d, -n)$

Corollary: $\text{DP} \subseteq \text{PMC}_0(\text{mcmp}, +c, -d, -n)$

⋆ New frontier of the efficiency in the framework $\text{DAM}_0(\ast, +c, -d, -n)$: non-cooperation in object evolution rules versus mcmp in object evolution rules.

Theorem: $\text{MAJORITY-SAT} \in \text{PMC}_0(\text{mcmp}, +c, -d, -n)$

Corollary: $\text{PP} \subseteq \text{PMC}_0(\text{mcmp}, +c, -d, -n)$

What about separation rules instead of division rules?

Theorem: $\text{PMC}_0(\text{SAM}_0(\text{mcmp}, +c, -d, +n)) = \text{P}$
Minimal cooperation and minimal production

Object evolution rules: \([ a \rightarrow b ]_h ; [ a b \rightarrow c ]_h\)

Theorem: \(\text{SAT} \in \text{PMC}^{DAM_0(\text{mcmp}, +c, -d, -n)}\).
Corollary: \(\text{DP} \subseteq \text{PMC}^{DAM_0(\text{mcmp}, +c, -d, -n)}\).

What about separation rules instead of division rules?

Theorem: \(\text{PMC}^{SAM_0(\text{mcmp}, +c, -d, +n)} = \mathbb{P}\).
**Minimal cooperation and minimal production**

**Object evolution rules:** $[a \rightarrow b]_h \land [a \ b \rightarrow c]_h$

**Theorem:** $\text{SAT} \in \text{PMC}_{\mathcal{DAM}^0(mcmp, +c, -d, -n)}$.

**Corollary:** $\text{DP} \subseteq \text{PMC}_{\mathcal{DAM}^0(mcmp, +c, -d, -n)}$.

* New frontier of the efficiency in the framework $\mathcal{DAM}^0(\ast, +c, -d, -n)$: **non-cooperation** in object evolution rules versus **mcmp** in object evolution rules.
Minimal cooperation and minimal production

Object evolution rules: \([ a \rightarrow b ]_h ; [ a b \rightarrow c ]_h\)

Theorem: \(\text{SAT} \in \text{PMC}^{\mathcal{DAM}^0(mcmp,+c,-d,-n)}\).

Corollary: \(\text{DP} \subseteq \text{PMC}^{\mathcal{DAM}^0(mcmp,+c,-d,-n)}\).

★ New frontier of the efficiency in the framework \(\mathcal{DAM}^0(\ast,+c,-d,-n)\): non-cooperation in object evolution rules versus \(\text{mcmp}\) in object evolution rules.

Theorem: \(\text{MAJORITY-SAT} \in \text{PMC}^{\mathcal{DAM}^0(mcmp,+c,-d,-n)}\).

Corollary: \(\text{PP} \subseteq \text{PMC}^{\mathcal{DAM}^0(mcmp,+c,-d,-n)}\).
Minimal cooperation and minimal production

Object evolution rules: $[ a \rightarrow b ]_h ; [ a b \rightarrow c ]_h$

Theorem: SAT $\in$ PMC\(\mathcal{D}\mathcal{A}\mathcal{M}^0(mcmp,+c,-d,-n)\).
Corollary: DP $\subseteq$ PMC\(\mathcal{D}\mathcal{A}\mathcal{M}^0(mcmp,+c,-d,-n)\).

★ New frontier of the efficiency in the framework \(\mathcal{D}\mathcal{A}\mathcal{M}^0(\ast,+c,-d,-n)\): non-cooperation in object evolution rules versus mcmp in object evolution rules.

Theorem: MAJORITY-SAT $\in$ PMC\(\mathcal{D}\mathcal{A}\mathcal{M}^0(mcmp,+c,-d,-n)\).
Corollary: PP $\subseteq$ PMC\(\mathcal{D}\mathcal{A}\mathcal{M}^0(mcmp,+c,-d,-n)\).

What about separation rules instead of division rules?
Minimal cooperation and minimal production

Object evolution rules: \[ [a \rightarrow b]_h ; [a b \rightarrow c]_h \]

**Theorem:** SAT $\in$ PMC$_{DAM^0(mcmp, +c, -d, -n)}$.

**Corollary:** DP $\subseteq$ PMC$_{DAM^0(mcmp, +c, -d, -n)}$.

★ New frontier of the efficiency in the framework $DAM^0(\ast, +c, -d, -n)$: 
**non-cooperation** in object evolution rules versus **mcmp** in object evolution rules.

**Theorem:** MAJORITY-SAT $\in$ PMC$_{DAM^0(mcmp, +c, -d, -n)}$.

**Corollary:** PP $\subseteq$ PMC$_{DAM^0(mcmp, +c, -d, -n)}$.

What about separation rules instead of division rules?

**Theorem:** PMC$_{SAM^0(mcmp, +c, -d, +n)} = P$. 
Counting membrane systems

**Decision problems**: abstract problem that has a *yes* or *no* answer.

- Recognizer membrane systems: The classes $\text{DAM}^0$ and $\text{SAM}^0$.

**Counting problems**: how many possible solutions exist associated with each instance.

- Counting membrane systems: inspired from recognizer membrane systems but the boolean answer of these systems is replaced by an *answer* encoded by a *natural number expressed in a binary notation*.
- The classes $\text{DAM}^0_c$ and $\text{SAM}^0_c$. 

\[ \text{Theorem:} \quad \#\text{SAT} \in \text{PMC}_{\text{DAM}^0_c}(\text{mcmp}, +c, -d, -n) \]

\[ \text{Corollary:} \quad \#\text{P} \subseteq \text{PMC}_{\text{DAM}^0_c}(\text{mcmp}, +c, -d, -n) \]
Counting membrane systems

Decision problems: abstract problem that has a yes or no answer.

- Recognizer membrane systems: The classes $DAM^0$ and $SAM^0$.

Counting problems: how many possible solutions exist associated with each instance.

- Counting membrane systems: inspired from recognizer membrane systems but the boolean answer of these systems is replaced by an answer encoded by a natural number expressed in a binary notation.
- The classes $DAM_C^0$ and $SAM_C^0$.

Theorem: $\#SAT \in PMC_{DAM_C^0}(mcmp,+c,−d,−n)$.

Corollary: $\#P \subseteq PMC_{DAM_C^0}(mcmp,+c,−d,−n)$. 
Counting membrane systems

**Decision problems**: abstract problem that has a *yes* or *no* answer.
- Recognizer membrane systems: The classes $\text{DAM}^0$ and $\text{SAM}^0$.

**Counting problems**: how many possible solutions exist associated with each instance.
- Counting membrane systems: inspired from recognizer membrane systems but the boolean answer of these systems is replaced by an *answer* encoded by a natural number expressed in a binary notation.
- The classes $\text{DAM}_C^0$ and $\text{SAM}_C^0$.

**Theorem**: $\#\text{SAT} \in \text{PMC}_{\text{DAM}_C^0}(\text{mcmp}, +c, -d, -n)$.

**Corollary**: $\#\text{P} \subseteq \text{PMC}_{\text{DAM}_C^0}(\text{mcmp}, +c, -d, -n)$.

What about the complexity class $\text{PMC}_{\text{SAM}_C^0}(\text{mcmp}, +c, -d, -n)$?
New results (IV)
New results (IV)

Theorem: \( \text{PMC}_{SAM^0}(mcmp,+c,-d,+n) = P \).
New results (IV)

Theorem: $\text{PMC}_{SAM^0}(mcmp, +c, -d, +n) = P$.

- New frontier of the efficiency in the framework $\mathcal{AM}^0(mcmp, +c, -d, -n)$: separation versus division.
New results (IV)

**Theorem:** $\text{PMC}_{S\mathcal{AM}^0(mcmp, +c, -d, +n)} = \text{P}$.  

- New frontier of the efficiency in the framework $\mathcal{AM}^0(mcmp, +c, -d, -n)$: separation versus division.

- New frontier of the efficiency in the framework $S\mathcal{AM}^0(mc, +c, -d, -n)$: mcmp versus pmc.
Minimal cooperation and minimal production in communication rules

\[
\begin{align*}
\text{mcmp in send-in communication rules} & : a[h] \rightarrow [b] h \rightarrow [c] h \rightarrow [a b] h \rightarrow [c] h \rightarrow [a b] h \\
\text{mcmp in send-out communication rules} & : [a] h \rightarrow [b] h \rightarrow [c] h \rightarrow [a b] h \rightarrow [c] h \rightarrow [a b] h
\end{align*}
\]

for \( h \in H \) and \( a, b, c \in \Gamma \)

The class \( \text{DAM}^{0} (+e, \beta, \pm d, \pm n) \), \( \beta \in \{ \text{mcmp in } - \text{out}, \text{mcmp in }, \text{mcmp out} \} \).
Minimal cooperation and minimal production in communication rules

- mcmp in send-in and send-out communication rules ($\text{mcmp}_{\text{in-out}}$):

  \[
  \begin{align*}
  a \, [ \, ]_h & \rightarrow [ \, b \, ]_h \\
  a \, b \, [ \, ]_h & \rightarrow [ \, c \, ]_h \\
  [ \, a \, ]_h & \rightarrow b \, [ \, ]_h \\
  [ \, a \, b \, ]_h & \rightarrow c \, [ \, ]_h
  \end{align*}
  \]

  for $h \in H$ and $a, b, c \in \Gamma$

- mcmp in send-in communication rules ($\text{mcmp}_{\text{in}}$):

  \[
  \begin{align*}
  a \, [ \, ]_h & \rightarrow [ \, b \, ]_h \\
  a \, b \, [ \, ]_h & \rightarrow [ \, c \, ]_h
  \end{align*}
  \]

  for $h \in H$ and $a, b, c \in \Gamma$

- mcmp in send-out communication rules ($\text{mcmp}_{\text{out}}$):

  \[
  \begin{align*}
  [ \, a \, ]_h & \rightarrow b \, [ \, ]_h \\
  [ \, a \, b \, ]_h & \rightarrow c \, [ \, ]_h
  \end{align*}
  \]

  for $h \in H$ and $a, b, c \in \Gamma$

The class $\mathcal{DAM}^0(\{+e, \beta, \pm d, \pm n\})$, $\beta \in \{\text{mcmp}_{\text{in-out}}, \text{mcmp}_{\text{in}}, \text{mcmp}_{\text{out}}\}$. 
New results

Theorem: SAT ∈ PMC_{\text{DAM}}(e, mcmp_{\text{in}} - mcmp_{\text{out}}, d, n).

Corollary: DP ⊆ PMC_{\text{DAM}}(e, mcmp_{\text{in}} - mcmp_{\text{out}}, d, n).

Direction in communication rules doesn't matter!!!

Theorem: SAT ∈ PMC_{\text{DAM}}(e, mcmp_{\text{in}}, d, n) ∩ PMC_{\text{DAM}}(e, mcmp_{\text{out}}, d, n).

Corollary: DP ⊆ PMC_{\text{DAM}}(e, mcmp_{\text{in}}, d, n) ∩ PMC_{\text{DAM}}(e, mcmp_{\text{out}}, d, n).

Are necessary division rules for non-elementary membranes?
New results

mcmp in communication rules (both directions):

Theorem: \( \text{SAT} \in \text{PMC}_{\text{DAM}}^{0}(+e, \text{mcmp}_{\text{in-out}}, -d, +n) \).

Corollary: \( \text{DP} \subseteq \text{PMC}(+e, \text{mcmp}_{\text{in}} - d, +n) \).

Direction in communication rules doesn’t matter!!!
New results

**mcmp in communication rules (both directions):**

**Theorem:** $\text{SAT} \in \text{PMC}_{\text{DAM}}(e, \text{mcmp}_{\text{in}} - \text{out}, -d, +n)$.

**Corollary:** $\text{DP} \subseteq \text{PMC}(e, \text{mcmp}_{\text{in}} - \text{out}, -d, +n)$.

**Simple object evolution rules:** $[a \rightarrow b]_h$, for $h \in H$ and $a, b \in \Gamma$.
New results

 mcmp in communication rules (both directions):

Theorem: $\text{SAT} \in \text{PMC}^{DAM}_0(+e, mcmp_{in-out}, -d, +n)$.

Corollary: $\text{DP} \subseteq \text{PMC}(+e, mcmp_{in-out}, -d, +n)$.

Simple object evolution rules: $[a \rightarrow b]_h$, for $h \in H$ and $a, b \in \Gamma$

Theorem: $\text{SAT} \in \text{PMC}^{DAM}_0(+e_s, mcmp_{in-out}, -d, +n)$.

Corollary: $\text{DP} \subseteq \text{PMC}(+e_s, mcmp_{in-out}, -d, +n)$.
New results

**mcmp in communication rules (both directions):**

**Theorem:** \( \text{SAT} \in \text{PMC}_{DAM}^0(+e,\text{mcmp}_{\text{in}}-\text{out},-d,+n) \).

**Corollary:** \( \text{DP} \subseteq \text{PMC}(+e,\text{mcmp}_{\text{in}}-\text{out},-d,+n) \).

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**Simple object evolution rules:** \([ a \rightarrow b ]_h\), for \( h \in H \) and \( a, b \in \Gamma \)

**Theorem:** \( \text{SAT} \in \text{PMC}_{DAM}^0(+e_{s},\text{mcmp}_{\text{in}}-\text{out},-d,+n) \).

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Direction in communication rules ...
New results

**mcmp in communication rules (both directions):**

**Theorem:** $\text{SAT} \in \text{PMC}_{DAM^0}(+e,\text{mcmp}_{\text{in-out}},-d,+n)$.

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**Theorem:** $\text{SAT} \in \text{PMC}_{DAM^0}(+e_s,\text{mcmp}_{\text{in-out}},-d,+n)$.

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**Direction in communication rules ... doesn’t matter!!!**
New results

**mcmp in communication rules (both directions)**:

**Theorem**: \( SAT \in \text{PMC}(e, \text{mcmp}_{\text{in}} - \text{out}, -d, +n) \).

**Corollary**: \( \text{DP} \subseteq \text{PMC}(e, \text{mcmp}_{\text{in}} - \text{out}, -d, +n) \).

**Simple object evolution rules**: \( [a \rightarrow b]_h \), for \( h \in H \) and \( a, b \in \Gamma \).

**Theorem**: \( SAT \in \text{PMC}(e, \text{mcmp}_{\text{in}} - \text{out}, -d, +n) \).

**Corollary**: \( \text{DP} \subseteq \text{PMC}(e, \text{mcmp}_{\text{in}} - \text{out}, -d, +n) \).

**Direction in communication rules** ... doesn’t matter!!!

**Theorem**: \( SAT \in \text{PMC}(e, \text{mcmp}_{\text{in}} - \text{out}, -d, +n) \cap \text{PMC}(e, \text{mcmp}_{\text{out}} - \text{in}, -d, +n) \).

**Corollary**: \( \text{DP} \subseteq \text{PMC}(e, \text{mcmp}_{\text{in}} - \text{out}, -d, +n) \cap \text{PMC}(e, \text{mcmp}_{\text{out}} - \text{in}, -d, +n) \).
New results

**mcmp in communication rules (both directions):**

Theorem: \( \text{SAT} \in \text{PMC}_{\text{DAM}^0}(+e,\text{mcmp}_{\text{in-out}},-d,+n) \cdot \)

Corollary: \( \text{DP} \subseteq \text{PMC}(+e,\text{mcmp}_{\text{in-out}},-d,+n) \cdot \)

**Simple object evolution rules:** \([a \rightarrow b]_h\), for \(h \in H\) and \(a, b \in \Gamma\)

Theorem: \( \text{SAT} \in \text{PMC}_{\text{DAM}^0}(+e_s,\text{mcmp}_{\text{in-out}},-d,+n) \cdot \)

Corollary: \( \text{DP} \subseteq \text{PMC}(+e_s,\text{mcmp}_{\text{in-out}},-d,+n) \cdot \)

**Direction in communication rules ... doesn’t matter!!!**

Theorem: \( \text{SAT} \in \text{PMC}_{\text{DAM}^0}(+e_s,\text{mcmp}_{\text{in}},-d,+n) \cap \text{PMC}_{\text{DAM}^0}(+e_s,\text{mcmp}_{\text{out}},-d,+n) \cdot \)

Corollary: \( \text{DP} \subseteq \text{PMC}(+e_s,\text{mcmp}_{\text{in}},-d,+n) \cap \text{PMC}_{\text{DAM}^0}(+e_s,\text{mcmp}_{\text{out}},-d,+n) \cdot \)

Are necessary **division rules for non-elementary membranes?**
References


THANK YOU
FOR YOUR ATTENTION!