Cell Division versus Membrane Fission:
A Computational Complexity Perspective

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13th Brainstorming Week on Membrane Computing
Sevilla, Spain, February 2, 2015

Cell division (I)

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One of the basic processes in the cell life cycle.

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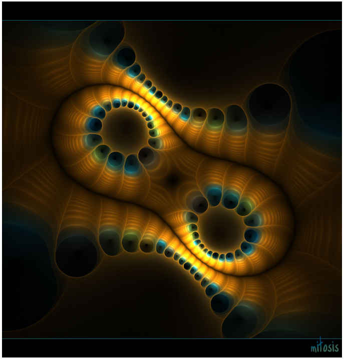
- *Binary fission* (prokaryotic cells)
- *Mitosis* (eukaryotic cells)
- *Meiosis* (eukaryotic cells)



Cell division (II)

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[3] Gh. Păun, M.J. Pérez-Jiménez, A. Riscos-Núñez. Tissue P system with cell division. *International Journal of Computers, Communications & Control* , **Vol. III, 3** (2008), 295–303.

Cell division (II)

Cell division inspired mechanism in Membrane Computing:

- P systems with active membranes (membrane division rules)[1]
  - ⋆ Evolution, Send-in, Send-out, Dissolution, Membrane division rules.
  - ⋆ Computational completeness.
  - ⋆ Computational efficiency (a semi-uniform polynomial time solution for SAT by using communication rules with length bounded by the number of clauses of the input[2]).

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- Tissue P systems with cell division (cell division rules)[3]
  - ⋆ Symport/antiport, Cell division rules.
  - ⋆ Computational completeness.
  - ⋆ Computational efficiency (a uniform polynomial time solution for SAT by using communication rules with length at most 5).

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Membrane fission (I)

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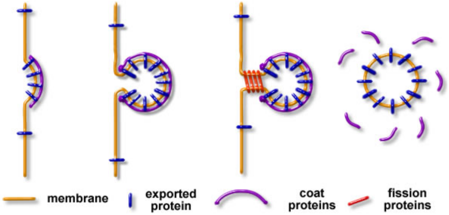
Lipid membranes:

- *Plasma membrane*
  - ⋆ Separates the interior of a cell from its environment.
- *Membrane compartments*
  - ⋆ Concentrations barriers alowing incorporate material (donnor-aceptor membrane).



Membrane fission (II)

[4] A. Alhazov, T.O. Ishdorj. Membrane operations in P systems with active membranes. In Gh.Păun et al. (eds.) *Proceedings of the Second Brainstorming Week on Membrane Computing*, Sevilla, 2-7 February 2004, Research Group on Natural Computing, TR 01/2004, University of Seville, 37-44.

[5] L. Pan, T.-O. Ishdorj. P systems with active membranes and separation rules. *Journal of Universal Computer Science*, **10**, 5 (2004), 630–649.

[6] L. Pan, M.J. Pérez-Jiménez. Computational complexity of tissue–like P systems. *Journal of Complexity*, **26**, 3 (2010), 296–315.

Membrane fission (II)

Membrane fission inspired mechanism in Membrane Computing:

- P systems with active membranes:
  - Membrane separation rules associated with subsets of the working alphabet[4] (a semi-uniform polynomial time solution to SAT by using evolution rules with length at most 5).
  - Membrane separation rules associated with a prefixed partition of the working alphabet[5] (computational completeness + a uniform polynomial time solution to SAT by using evolution rules with length at most 5).
- Tissue P systems with cell separation:
  - Cell separation rules associated with a prefixed partition of the working alphabet[6] (a semi-uniform polynomial time solution to SAT by using evolution rules with length at most 8).

[4] A. Alhazov, T.O. Ishdorj. Membrane operations in P systems with active membranes. In Gh.Păun et al. (eds.) *Proceedings of the Second Brainstorming Week on Membrane Computing*, Sevilla, 2-7 February 2004, Research Group on Natural Computing, TR 01/2004, University of Seville, 37-44.

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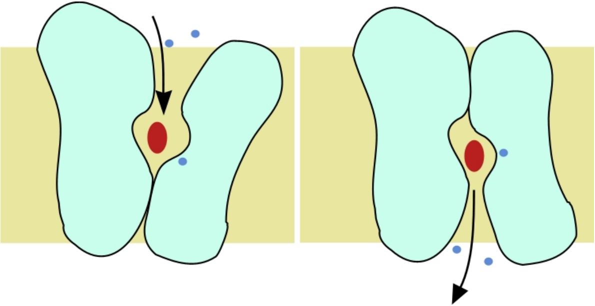
Trans-membrane transport

[7] A. Păun, Gh. Păun, G. Rozenberg. Computing by communication in networks of membranes, *International Journal of Foundations of Computer Science*, **13**, 6 (2002), 779–798.

Trans-membrane transport

Networks of membranes which compute by communication only:

- Symport/antiport rules[7] .
- Used both for communication with the environment and for direct communication between membranes.
- The environment plays an active role.
- Computational completeness.



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Cell-like P systems with symport/antiport rules

[8] A. Păun, Gh. Păun. The power of communication: P systems with symport/antiport, *New Generation Computing*, **20**, 3 (2002), 295–305.

Cell-like P systems with symport/antiport rules

$\Pi = (\Gamma, \mathcal{E}, \Sigma, \mu, \mathcal{M}_1, \ldots, \mathcal{M}_q, \mathcal{R}_1, \cdots, \mathcal{R}_q, i_{in}, i_{out})$ [8]

- ⋆ Computational completeness.

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Cell-like versus Tissue-like: Symport/antiport rules:

	Cell-like	tissue-like
Set of rules	Each membrane has associated a set of rules	Associated with the system
Structure	Rooted tree: defined in an explicit way	Directed graph: defined by the set of rules
Environment	Only skin membrane can communicate with it	Any cell can communicate with it
Communication	Two membranes: in an indirect way	Two cells: directly

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For each $k \geq 1$, we consider the following classes of recognizer cell-like P systems (**set of rules associated with each membrane**)

$\boxed{\mathcal{CDC}(k)}$

- $(u, out; v, in)$, for $u, v \in \Gamma^*$ (*symport-antiport rules*) whose length $(|u| + |v|)$ is at most k.
- $[\ a\ ]_i \rightarrow [\ b\ ]_i[\ c\ ]_i$, where $i \in \{1, 2, \ldots, q\}$ and $a, b, c \in \Gamma$ (*division rules*).

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The semantics is similar to the tissue P systems with cell division or separation.

Cell-like P systems with symport/antiport rules and "without environment"

$\Pi = (\Gamma, \mathcal{E}, \Sigma, \mu, \mathcal{M}_1, \ldots, \mathcal{M}_q, \mathcal{R}_1, \cdots, \mathcal{R}_q, i_{in}, i_{out})$ such that $\mathcal{E} = \emptyset$.

- ⋆ No objects initially located in the environment of the system available in an arbitrary number of copies.
- ⋆ In such P systems objects in the environment always have finite multiplicity.

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The classes $\widehat{\mathcal{CDC}(k)}$ and $\widehat{\mathcal{CSC}(k)}$

Recent results

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- $\mathbf{P} = \mathbf{PMC}_{\widehat{\mathcal{CSC}}}$ (algorithmic technique).
- For each $k \geq 1$, $\mathbf{PMC}_{\mathcal{CDC}(k)} = \mathbf{PMC}_{\widehat{\mathcal{CDC}(k)}}$ (simulation technique).

Proof techniques

Dependency graph

- Construction of a directed graph (*dependency graph*) G_Π associated with a P system Π verifying:
  - There exists an accepting computation of Π if and only if there exists a path between two distinguished nodes in the dependency graph associated with it.

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Simulation

- ⋆ Π' *simulates* Π *in an efficient way* if the following holds:

  (a) Π' can be constructed from Π by a DTM working in polynomial time.

  (b) There exists an injective function, f, from $\textbf{Comp}(\Pi)$ onto $\textbf{Comp}(\Pi')$ such that:

  - There exists a DTM that constructs $f(\mathcal{C})$ from computation $\mathcal{C}$ in polynomial time.
  - A computation $\mathcal{C}$ is an accepting computation if and only if $f(\mathcal{C})$ is an accepting one.
  - There exists a polynomial function $p(n)$ verifying $|f(\mathcal{C})| \leq p(|\mathcal{C}|)$ for each $\mathcal{C} \in \textbf{Comp}(\Pi)$.

Frontiers of the efficiency

NonEfficiency *(Feasible)*	*Efficiency* *(Presumably Efficient)*	
$\mathcal{CDC}(1)$	$\mathcal{CDC}(2)$	(*length*)
$\mathcal{CSC}(2)$	$\mathcal{CSC}(3)$	(*length*)
$\mathcal{CSC}(2)$	$\mathcal{CDC}(2)$	(*kind*)
$\widehat{\mathcal{CSC}(2)}$	$\widehat{\mathcal{CDC}(2)}$	(*kind*)
$\widehat{\mathcal{CSC}}$	$\mathcal{CSC}$	(*environment*)

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$\widehat{\mathcal{CSC}}$	$\mathcal{CSC}$	*(environment)*

Each such frontier provides a new way to tackle the P versus NP problem.

Does structure matter?

Similar results for:

- Tissue P systems with symport/antiport rules (cell division/cell separation).
- Cell-like P systems with symport/antiport rules (cell division/cell separation).

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Open questions:

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2. **Tissue** P systems with symport/antiport rules, can be efficiently simulated by **cell-like** P systems with symport/antiport rules?

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Idea: Complexity aspects on **Tissue P systems with active cells**.