Continuous Versus Discrete: Some Topics with a Regard to Membrane Computing

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Summary. Some questions and open problems are formulated in the context of a dilemma continuous approach versus discrete approach to the investigations of dynamics of complex biological and physical systems with a regard to membrane computing [11].

1 A question about an extent of discretization programs of physics

Fredkin–Sorkin–Wolfram discretization programs of physics via E. Fredkin's digitalization [5], R. D. Sorkin's causal sets [15], and S. Wolfram's cellular automata approach [18] give rise to a question:

Does the discretization mean a lost (or eventually how to find or establish counterparts) of classical qualitative properties of continuously (with respect to time among others) treated processes like the properties:

- a property of reaching equilibrium and its stability [16],
- asymptotic behaviour (i.e. tending of process trajectories—the solutions of some differential equations to some possibly regular curves like limiting cycles [16]),
- irregular behaviour:
  - chaos [6], [7], [16], [13], [14], [1], [2],
  - perturbations and noise approached by stochastic treatment of system dynamics.

One should notice that the status of the concepts of an equilibrium and its stability varies from biomedical physist's critique that these concepts are not adequate to capture creative forces of nature—"a system that reached equilibrium is 'dead'", cf. [7], to their importance, for instance, for the methods (due to G. Grossberg and J. J. Hopfield) of modelling (associative) memory and learning (training) in neural networks, reviewed, e.g., in [9].

The varying status of the concepts of an equilibrium and its stability accompanied the emergence of a new research area, called *nonlinear science* (cf. [13], [14]), comprising nonlinear dynamics (cf. Box1 in [17] and the book [16]), where chaos is an important issue.

Nonlinear science requires new mathematical tools beyond calculus and the discretization mentioned above provided some of these new tools, e.g. cellular automata.

The discretization does not diminish the role of continuous-time approach to system dynamics. The review [4] and the papers [1], [2] confirm that the continuous-time approach is still alive.

2 Answer

Some (partial) answer to the main question of Section 1 is contained in:

- characterization of irregular behaviour of processes represented by large graphs (like causal sets and their Hasse diagrams) and networks in terms of dimensions [10], in particular fractal dimension [12], like chaos in continuous dynamics is approached in terms of fractals [16];
- the attempts of making the discrete constructs continuous one, like K. Martin and P. Panangaden work [8] of building back space-time manifold from Sorkin like causal order;
- the embeddings of discrete-time system behaviour in continuous-time dynamics, cf. [4], where an embedding of a Turing machine behaviour in continuous-time dynamics is presented.

Concerning membrane computing [11] one could:

- represent processes generated by P systems by causal sets like T. Bolognesi [3] represents computational processes of various mechanisms,

then

- approach the causal sets representing processes generated by P systems like in the answer to the main question given above.

One could also investigate P system behaviour by its embedding in continuous-time dynamics, like in [4], to approach the irregularities, like chaos, of the resulting continuous-time dynamics of P systems in the manner of [1], [2].

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