Spiking Neural P Systems and Modularization of Complex Networks from Cortical Neural Network to Social Networks

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Summary. An idea of modularization of complex networks (from cortial neural net, Internet computer network, to market and social networks) is explained and some its topic motivations are presented. Then some known modularization algorithms and modular architectures (constructions) of complex networks are discussed in the context of possible applications of spiking neural P systems in order to improve these modularization algorithms and to analyze massively parallel processes in networks of modular architecture.

1 Introduction

The aim of this paper is to discuss certain interconnections between spiking neural P systems [16], [28], and an idea of modularization of complex networks from cortial neural net, Internet computer network, to market and social networks, where the idea of modularization comprises modular architectures (structures or constructions) of those networks and modularization algorithms for retrieving modular structure of networks. The interconnections are understood here as proposals of application of spiking neural P systems to improve the modularization algorithms and to analyze massively parallel processes in networks of modular architecture or construction (emergence of new modules, etc., [9], [23]).

In Section 2 we explain the idea of modularization of complex networks and some its topic motivations. In Section 3 we outline open problems concerning improvement of some modularization algorithms by application of spiking neural P systems and investigations of massively parallel processes in networks of modular construction by applying these P systems.

2 Modularization of Complex Networks and Its Topic Motivations

A *modularization*[1] of a complex network or a graph is understood to be a decomposition or a partition of the underlying set of nodes of the network or the graph, respectively, into subsets called *modules*, often identified with subnetworks determined by these subsets and treated as autonomous processing units in cooperation with other units (on a higher level if abstraction). A collection of modules of a given network or a graph can be also a subject of modularization, i.e. a decomposition into subcollections of modules, etc., where the resulting subcollections of modules are called *higher level modules.*

There are many reasons, motivations, and practical applications of modularization and we outline here some topics:

1) cortial neural network is modularized from anatomical, physiological, and scale (or magnitude) reasons, see, e.g., [24] or [27] for more references, into
   - cortial minicolumns which are modules consisting of neurons,
   - cortial hypercolumns which are some sets of minicolumns,
   - cortial areas which are some sets of hypercolumns,

   where cortial hypercolumns and areas are higher level modules,
2) natural self-modularization of cortial neural network into neuronal groups during evolution process described by M. G. Edelman's Theory of Neuronal Group Selection (Neuronal Darwinism), see [18], [17] for a spiking neural network version,
3) modularization of cortial neural network into assemblies of neurons appears useful for neuronal representation of cognitive functions and processes because:
   - a single neuron behavior is less certain or more noisy than a behavior of an assembly of neurons,
   - the number of synaptic connections of a single neuron with other neurons is smaller than that of an assembly of neurons with other assemblies of neurons,
   - according to M. Kaiser [20] hierarchical cluster (higher level module) architecture "may provide the structural basis for the stable and diverse functional patterns observed in cortial networks",
4) emergence (or extraction) of community structures in social networks, biological networks, and Internet computer network is a modularization of these networks discussed by M. J. E. Newman, [6], [7], [26], [29], see also applications of similar modularization in city planning discussed by Ch. Alexander [2],
5) modularization of artificial cortial-like networks for image processing, e.g., regularization for improving segmentation, see J. A. Anderson and P. Sutton, cf. [21], applications of an idea of a Network of Networks (NoN),
6) modularization which gives rise to hierarchical and fractal graphs and networks, [25], [29], [30], [33], to compress the information contained in large complex networks.

[1] The term 'modularization' is used e.g., in [19].

The higher level modules and their motivation are also discussed in [10], [11], [12], [22], [32], [31].

3 Modularization Algorithms and Modular Architectures

In the cases 2)–5) algorithms of modularization are considered, i.e. algorithms of distinguishing or extraction of modules, see e.g., [3], [18]. Thus one asks for those spiking neural P systems which could realize these algorithms through massive parallelism of computations providing

- efficiency of computation,
- those computation processes which could be close (from simulation reason) to real distributed processes of emergence of neuronal groups (see [18]) or community structures in social networks, where distributed processes of emergence of neuronal groups are massively parallel processes of simultaneous emergence of many those groups which are autonomous understood that, e.g., each group has at least one neuron which does not belong to other groups.

These spiking neural P systems could give rise to constructing new brain-based devices (robots) similar to those which belong to the family Darwin due to M.G. Edelman [8]. The new brain-based devices could simulate maturing processes, where emergence of neuronal groups and groups of groups give rise to new cognitive functions.

We show now an example of a link between modularization algorithms and spiking neural P systems which suggests the proposed above applications of these P systems. Namely, basing on the algorithm for identification of neuronal groups described in [18] we outline a method of extraction of a process of simultaneous emergence of many neuronal groups from a process generated by a spiking neural P system.

Let $\mathcal{S}\ [\ \mathcal{C} > \mathcal{S}'$ be the next state relation determined by simultaneous application (in maximal parallelism mode) of the rules of a spiking neural P system, where $\mathcal{S}, \mathcal{S}'$ are spike contents of neurons of the system and $\mathcal{C}$ is the set of those synapses of the system which are activated to transform $\mathcal{S}$ into $\mathcal{S}'$ according to some maximal consistent set of the rules of the system during a unit of time. For a finite process generated by a spiking neural P system and represented by

$$\mathcal{S}_0\ [\ \mathcal{C}_1 > \mathcal{S}_1\ [\ \mathcal{C}_2 > \mathcal{S}_2 \ldots \mathcal{S}_{n-1}\ [\ \mathcal{C}_n > \mathcal{S}_n \quad (n > 2) \tag{1}$$

we extract from it a process of simultaneous emergence of many neuronal groups which is represented by a sequence $\mathcal{G}_1 \ldots \mathcal{G}_{i^*}$ of sets of synapses of the system such that

- $\mathcal{G}_1$ is the set of maximal (with respect to inclusion relation of sets) subsets x of $\mathcal{C}_1$ such that the synapses in x have a common source neuron which is a counterpart of an anchor neuron, see the first step of the algorithm in [18],

- for $i > 1$ we define $\mathcal{G}_i$ to be the set of maximal sets in the collection

$$\mathcal{K}_i = \Big\{ y \;\Big|\; x \subsetneq y = x \cup \{ s \in \mathcal{C}_i \,|\, \text{the source neuron of synapse } s \\ \text{is the target neuron of some synapse in } x \} \text{ for some } x \in \mathcal{G}_{i-1} \Big\}$$

  until this collection is non-empty or, equivalently, until $i = i^*$, where i^* is the greatest number for which $\mathcal{K}_{i^*}$ is non-empty.

The elements of $\mathcal{G}_{i^*}$ represent neural circuits which correspond to neuronal groups emerging simultaneously in the process represented by (1).

Since the networks and their modularization discussed in 2)–4) are also approached by using probabilistic and statistical methods of clustering (see [1]) and by using random graphs (understood as in the B. Bolobas book [4]), it is worth to discuss a concept of a stochastic (or random) spiking neural P system whose synaptic connections form a random graph.

Besides the new applications of spiking neural P systems suggested above one could ask for an application of M.-A. Guttiérez-Naranjo and M. Pérez-Jiménez models for Hebbian learning with spiking neural P systems [15] to explain in a new way

- temporal correlation hypothesis of visual feature integration [14], also dealing with modularization, where modules are neural assemblies emerging in distributed processes like the processes of emergence of neuronal groups described above, for the connections of neuronal groups and binding (some generalization of feature integration), see [17].
- emergence of brain cognitive functions, where some modularizations of cortial neural network are considered, see [5], [13].

We propose to use higher-level networks with neighbourhood graphs, introduced in [27], as a precise description of results of some modularizations of networks and modular architectures including higher level modules.

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