Image Thresholding with Cell-like P Systems

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Summary. P systems are a new class of distributed parallel computing models. In this paper, a novel three-level thresholding approach for image segmentation based on cell-like P systems is proposed in order to improve the computational efficiency of multi-level thresholding. A cell-like P system with a specially designed membrane structure is developed and an improved evolution mechanism is integrated into the cell-like P system. Due to parallel computing ability and particular mechanism of the cell-like P system, the presented thresholding approach can effectively search the optimal thresholds for three-level thresholding based on total fuzzy entropy. Experimental results of both qualitative and quantitative comparisons for the proposed approach and GA-based and PSO-based approaches illustrate the applicability and effectiveness.

Key words: Image segmentation, Thresholding approach, Membrane computing, Cell-like P systems, Total fuzzy entropy

1 Introduction

Membrane computing, as a new branch of natural computing, was proposed by Păun [1] in 2000. Membrane computing is a novel class of distributed parallel computing models, which is inspired by the structure and functioning of living cells, as well as from the way the cells are organized in tissues or higher order structure [2]. The computing models are commonly called P systems. Since then, a large number of P systems and their variants have been proposed [3, 4, 5, 6, 7, 8, 9, 10]. The main ingredients of a P system are (i) *the membrane structure*, delimiting compartments where (ii) *multisets of objects* evolve according to (iii) *(reaction) rules* of a bio-chemical inspiration. According to their structures, these models can be divided into three categories: cell-like P systems, tissue-like P systems and neural-like P systems. The investigation of P systems mainly focuses on building a variety of computing models for different classes of problems, such as computing power, computational efficiency, and so on. The application research of P systems, especially applying P systems to solve real-world problems, has been concerned about in recent years. Among them, membrane algorithms are a class of representative models, which have been successfully used to deal with optimization problems [11, 12], control problems [13] and signal processing [14]. This paper focuses on application of P systems to image segmentation problem.

Image segmentation is a process of grouping an image into units that are homogeneous with respect to one or more characteristics. It is an important task in image analysis. Thresholding is widely used as a popular technique in image segmentation. The goal of thresholding is to separate objects from background image or discriminate objects from objects that have distinct gray levels. Over these years, A large number of thresholding techniques have been addressed [15, 16, 17]. Bi-level thresholding, which is firstly discussed, segments an image into two different regions. The pixels with gray values greater than a certain threshold are classified as object pixels, and the others with gray values lesser than the threshold are classified as background pixels. Otsu's approach [18] and Kapur's approach [19], which find the optimal thresholds by maximizing the between-class variance of gray levels and the entropy of the histogram respectively, are simple and effective in bi-level thresholding. However, the gray level histograms of most of the images in the real world is multimodal. Therefore, multi-level thresholding has been received many attentions in recent years. Multi-level thresholding determines more than one threshold for an image and segments the image into several distinct regions, which corresponds to one background and several object. The Otsu' and Kapur's approaches can be extendable to multi-level thresholding but inefficient in determining the optimal thresholds due to the exponential growth in computation time. To improve the efficiency, some approaches have been proposed to reduce the computational complexity of determining the multi-level thresholds, such as the recursive algorithm [20]. But they still suffer from long processing time when the number of thresholds increases. The fuzzy entropy has been introduced into image segmentation in recent years [21, 22, 23, 24]. Cheng et al. [21] proposed a thresholding approach, where the fuzzy relation and the maximum fuzzy entropy were used to perform fuzzy partition on a two-dimensional histogram. In [22], Shelokar et al. found the optimal threshold by minimizing the sum of the fuzzy entropies. Zhao et al. [23] presented a three-level thresholding approach based on fuzzy entropy. In [24], Liu et al. presented a fuzzy classification entropy to deal with multi-level thresholding. However, these approaches still suffer from the same problem mentioned above. In order to overcome this problem, some intelligent computing approaches have been applied to solve multi-level thresholding problems, such as genetic algorithm (GA), particle swarm optimization (PSO) and ant colony optimization (ACO). Yin et al. [25] presented a GA-based thresholding approach, where the objective function was similar to Otsu's or Kapur's function. In [26], Cheng et al. defined an approach to fuzzy entropy and employed the GA to find the optimal combination of the fuzzy parameters. Tao et al. [27] presented a three-level thresholding approach that uses the GA to find the optimal thresholds by maximizing the fuzzy entropy. In [28], Hammouche et al. proposed a multi-level thresholding approach, which allows the determination of the appropriate number of thresholds as well as the adequate threshold values. However, GA has some drawbacks such as slow convergence rate, premature convergence to local minima. Thus, the PSO has been applied to multi-level thresholding [29, 30, 31]. In addition, Tao et al. [32] used the ACO to obtain the optimal parameters of the presented entropy-based object segmentation approach. Currently, adapting P systems to solve image segmentation problems has been addressed [33, 34]. Díaz-Pernil et al. [33] combined the membrane structure and symport-antiport communication rules of tissue-like P systems to deal with homology groups of binary 2D image. Wang et al. [34] presented a bi-level image thresholding approach.

In this paper, we propose a novel three-level thresholding approach based on cell-like P systems for image segmentation. Our main motivation is to improve and enhance the efficiency of multi-level thresholding approach based on the fuzzy entropy criterion by applying the parallel computing ability as well as specially designed structure and mechanisms of cell-like P systems. The proposed three-level thresholding approach is evaluated on several standard images and compared with the GA-based and PSO-based approaches.

The rest of this paper is organized as follows. Section 2 briefly describes the maximum fuzzy entropy principle. The proposed three-level thresholding approach based on cell-like P systems is presented in Section 3. Experimental results are provided in Section 4. Finally, Section 5 draws the conclusions.

2 Maximum Fuzzy Entropy Principle

In this section, we briefly review maximum fuzzy entropy principle and give fuzzy membership functions used in this paper.

Let $D = \{(i,j) \mid i = 0,1,\dots,M-1; j = 0,1,\dots,N-1\}$, $G = \{0,1,\dots,l-1\}$, where M, N and l are three positive integers. Let $I(x,y)$ be the gray level of an image I at the pixel (x,y). Denote

$$D_k = \{(x,y) \mid I(x,y) = k,\ (x,y) \in D\},\ k = 0,1,2,\dots,l-1 \tag{1}$$

$$h_k = \frac{n_k}{M \times N} \tag{2}$$

where n_k is the number of pixels in D_k. So, $0 \le h_k \le 1$, $\sum_{n=0}^{l-1} h_k = 1$. Let $H = \{h_0, h_1, \dots, h_{l-1}\}$ be the gray histogram of the image I. $\{D_o, D_1, \dots, D_{l-1}\}$ forms a probability partition of D and its probabilistic distribution is $p_k = P(D_k) = h_k$ $(k = 0,1,\dots,l-1)$.

In this paper, we will deal with three-level image thresholding, which has two thresholds, t_1 and t_2. The two thresholds will segment the image I into three gray levels, low gray level, middle gray level and high gray level, and the corresponding domains are denoted by D_l, D_m and D_h respectively. Thus, $D = D_l \bigcup D_m \bigcup D_h$, $D_l \bigcap D_m = D_l \bigcap D_h = D_m \bigcap D_h = \phi$. Let p_l, p_m and p_h be the probabilistic distributions of D_l, D_m and D_h respectively, i.e., $p_l = P(D_l)$, $p_m = P(D_m)$, $p_h = P(D_h)$. However, these probabilistic distributions are unknown.

For $k = 0, 1, \ldots, 255$, denote

$$\begin{aligned} D_{kl} &= \{(x,y) \mid I(x,y) \leq t_1,\ (x,y) \in D_k\} \\ D_{km} &= \{(x,y) \mid t_1 < I(x,y) \leq t_2,\ (x,y) \in D_k\} \\ D_{kh} &= \{(x,y) \mid I(x,y) > t_2,\ (x,y) \in D_k\} \end{aligned} \tag{3}$$

Then, we have

$$\begin{aligned} p_{kl} &= P(D_{kl}) = p_k \times p_{l|k} \\ p_{km} &= P(D_{km}) = p_k \times p_{m|k} \\ p_{kh} &= P(D_{kh}) = p_k \times p_{h|k} \end{aligned} \tag{4}$$

with a constraint that $p_{l|k} + p_{m|k} + p_{h|k} = 1$ $(k = 0, 1, \ldots, 255)$. Thus, $p_l = \sum_{k=0}^{255} p_k \times p_{l|k}$, $p_m = \sum_{k=0}^{255} p_k \times p_{m|k}$ and $p_h = \sum_{k=0}^{255} p_k \times p_{h|k}$.

Let $\mu_l(k)$, $\mu_m(k)$ and $\mu_h(k)$ denote the membership grades of a pixel belonging to D_l, D_m and D_h respectively. Then

$$p_l = \sum_{k=0}^{255} p_k \times \mu_l(k),\ p_m = \sum_{k=0}^{255} p_k \times \mu_m(k),\ p_h = \sum_{k=0}^{255} p_k \times \mu_h(k) \tag{5}$$

In this paper, we employ the following three functions to approximate the membership functions $\mu_l(k)$, $\mu_m(k)$ and $\mu_h(k)$, respectively

$$\mu_l(k) = \begin{cases} 1, & k \leq a \\ 1 - \frac{(k-a)^2}{(c-a)\times(b-a)}, & a < k \leq b \\ \frac{(k-c)^2}{(c-a)\times(c-b)}, & b < k \leq c \\ 0, & k > c \end{cases} \tag{6}$$

$$\mu_m(k) = \begin{cases} 0, & k \leq a \\ \frac{(k-a)^2}{(c-a)\times(b-a)}, & a < k \leq b \\ 1 - \frac{(k-c)^2}{(c-a)\times(c-b)}, & b < k < c \\ 1, & k = c \\ 1 - \frac{(k-c)^2}{(e-c)\times(d-c)}, & c < k \leq d \\ \frac{(k-e)^2}{(e-c)\times(e-d)}, & d < k \leq e \\ 0, & k > e \end{cases} \tag{7}$$

$$\mu_h(k) = \begin{cases} 0, & k \leq c \\ \frac{(k-c)^2}{(e-c)\times(d-c)}, & c < k \leq d \\ 1 - \frac{(k-e)^2}{(e-c)\times(e-d)}, & d < k \leq e \\ 1, & k > e \end{cases} \tag{8}$$

where $0 < a \leq b \leq c \leq d \leq e < 255$.

The fuzzy entropies of above three classes are given as follows:

$$\begin{aligned} H_l &= -\sum_{k=0}^{255} \frac{p_k \times \mu_l(k)}{p_l} \times \ln\left(\frac{p_k \times \mu_l(k)}{p_l}\right) \\ H_m &= -\sum_{k=0}^{255} \frac{p_k \times \mu_m(k)}{p_m} \times \ln\left(\frac{p_k \times \mu_m(k)}{p_m}\right) \\ H_h &= -\sum_{k=0}^{255} \frac{p_k \times \mu_h(k)}{p_h} \times \ln\left(\frac{p_k \times \mu_h(k)}{p_h}\right) \end{aligned} \tag{9}$$

Then the total fuzzy entropy is computed by

$$H(a, b, c, d, e) = H_l + H_m + H_h \tag{10}$$

From the Eq.(10), we can see that H is the function of five parameters a, b, c, d, e in fact. The optimal image thresholding is to find the most appropriate combination of these parameters so that the total fuzzy entropy $H(a, b, c, d, e)$ achieves the maximum value. Then the most appropriate combination of these parameters, by which the image I is segmented into three classes, can satisfy the following relation:

$$\mu_l(t_1) = \mu_m(t_1) = 0.5,\ \ \mu_m(t_2) = \mu_h(t_2) = 0.5 \tag{11}$$

Note that threshold t_1 is the intersection point of curves $\mu_l(k)$ and $\mu_m(k)$, while threshold t_2 is the intersection point of curves $\mu_m(k)$ and $\mu_h(k)$. Therefore, according to Eqs.(6)-(8), the two thresholds can be determined as follows:

$$t_1 = \begin{cases} a + \sqrt{(c-a) \times (b-a)/2}, & (a+c)/2 \leq b \leq c \\ c - \sqrt{(c-a) \times (c-b)/2}, & a \leq b \leq (a+c)/2 \end{cases} \tag{12}$$

$$t_2 = \begin{cases} c + \sqrt{(e-c) \times (d-c)/2}, & (c+e)/2 \leq d \leq e \\ e - \sqrt{(e-c) \times (e-d)/2}, & c \leq d \leq (c+e)/2 \end{cases} \tag{13}$$

3 The Proposed Image Thresholding Approach

The proposed thresholding approach is based on a cell-like P system. In order to effectively deal with three-level thresholding problem under P systems, we design a special membrane structure with three layers, which consists of $(2m+1)$ membranes, shown in Fig. 1. These membranes are labeled by $1, 2, \ldots, m, m+1, m+2, \ldots, 2m, 2m+1$, respectively. The m membranes labeled by $1, 2, \ldots, m$, which are called evolution membranes, will cooperatively evolve the objects in the system to find the optimal segmentation thresholds. Each evolution membrane has one child membrane, called local store membrane, whose role is to store the best object found as far in the evolution membrane. In Fig.1, membranes $m+1, m+2, \ldots, 2m$ are the local store membranes of evolution membranes $1, 2, \ldots, m$, respectively. In each computing step, if a evolution membrane find its new best object by evolution rule it will transmit the new best object into the corresponding local store membrane and skin membrane $(2m+1)$. The skin membrane is called global store membrane in this paper, whose role is to store best object found as far in entire system. In the beginning of computing step, each evolution membrane will receive local best object from the corresponding local store membrane as well as global best object from global store membrane (skin membrane). In Fig.1, the arrows with different directions indicate the transitive relations of objects. As usual in P systems, these evolution membranes as parallel computing units work in a maximally parallel way (a universal clock is considered here).

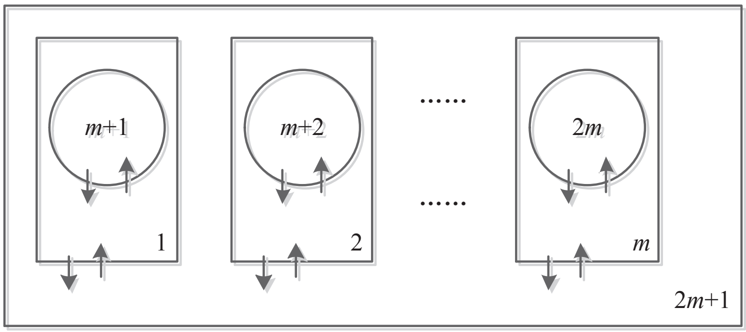

Fig. 1. Membrane structure of the used cell-like P system.

As we known, every membrane contains a certain number of objects. For simplicity, we assume that every evolution membrane contains same number of objects, and the number is denoted by n. However, local store membranes and global store membrane contain only one object respectively. In this work, each object is a five-dimensional vector $X = (x_1, x_2, x_3, x_4, x_5)$, where x_1, x_2, x_3, x_4 and x_5 correspond to five segmentation parameters, a, b, c, d, and e respectively. Therefore, each object in fact expresses a candidate of the optimal segmentation thresholds to be found. In the cell-like P system, the total fuzzy entropy (i.e., Eq. (10)) will be regarded as fitness function of object in the system to evaluate the quality of each object, i.e., $Fitness = H(a, b, c, d, e)$. Initially, we randomly generate n objects for each evolution membrane and fill its best object into the corresponding local store membrane, and then fill the best object of entire system into global store membrane. Note that if a randomly generate object do not hold the increasing order $0 < a \leq b \leq c \leq d \leq e < 255$, we re-compute its components as follows:

$$\begin{cases} c' = c \\ b' = c' \times (b/255) \\ a' = b' \times (a/255) \\ d' = c' + (255 - c') \times (d/255) \\ e' = d' + (255 - d') \times (e/255) \end{cases} \tag{14}$$

In this work, multiple evolution membranes are designed to collaboratively evolve objects in the system, thus this will accelerate the exploitation of optimal segmentation parameters. The mutation operation and crossover operation of differential evolutionary (DE) algorithm are used as evolution rules of evolution membranes, however, we use a modified mutation operation according to special structure of the cell-like P system, which can be viewed as a variant of the rule "DE/current-to-best/1" in the DE . During one computing step, each evolution membrane will use the modified mutation operation to generate a mutation object for its every current object, X_i,

$$Y_i = X_i + F \cdot (X_{lbest} - X_i) + F(X_{gbest} - X_i) + F(X_{r_1} - X_{r_2}), \tag{15}$$

where X_{lbest} is the best object form the corresponding local archive membrane, X_{gbest} is the best object from global archive membrane, and X_{r_1}, X_{r_2} are two randomly selected objects from current objects. The scaling factor F is a positive control parameter for scaling the difference vector. The improved mutation rule is based on the point: two best objects, which are from different sources (local and global store membranes), will guide the evolution of objects and speed up the convergence, and can also improve the diversity of objects in the system.

After the mutation operation, crossover operation is applied to each pair of the current object and its corresponding mutant object to generate a trial object Z_i, and the crossover operation is defined as follows:

$$Z_i = \begin{cases} Y_i, \text{ if } \mathrm{rand}_i \leq \ CR \text{ or } j = \mathrm{rand}_j \\ X_i, \text{ otherwise} \end{cases} \tag{16}$$

where the crossover rate CR is a user-specified constant within the range $[0, 1]$, which controls the fraction of parameter values copied from the mutant object, and rand_j is a randomly chosen integer in the range $[1, 5]$.

The maximum execution step number is employed as halt condition in the proposed three-level thresholding approach based on cell-like P systems. When the system halts, the object in the skin membrane is regarded as the output of entire system.

The proposed three-level thresholding approach based on cell-like P systems is summarized as follows.

Three-level Thresholding Approach Based on Cell-like P Systems

```
program Three-level-thresholding
   Input:
```

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  Number of evolution membranes m;
  Number of objects in each evolution membrane n;
  Maximum execution step number Smax;
  Scaling factor F;
  Crossover rate CR;
Output:
  Optimal thresholds (a,b,c,d,e);
begin
  Step 1: /* Initialization */
    for k=1 to m
       for j=1 to n
         /* Generate initial objects for each evolution
            membrane */
         X(k,j) = rand(5,255);
         /* Calculate the fitness value of the object
            according to Eq.(10) */
         Fit(k,j) = FitnessCalculation(X(k,j));
       end for
    end for
    Set computing step s = 0;
  Step 2:  /* Object evolution in membranes */
    Receive best object from global store membrane;
    for each evolution membrane k in parallel do
       Receive best object from its local store membrane;
       for j=1 to n
         Evolve the object X(k,j) in the evolution membrane k
            according to Eqs.(15)-(16);
         Fit(k,j) = FitnessCalculation(X(k,j));
       end for
       /* Update best object into its local store membrane and
          global store membrane */
       Update Xlbest(k) and Xgbest(k);
    end for
  Step 3: /* Halt condition judgment */
    If s > Smax is satisfied then
       Export object in skin membrane as (a,b,c,d,e);
       HALT;
    else
       s = s + 1;
       goto Step 2;
    end if
end.
```

4 Experimental results

The applicability and efficiency of the proposed image thresholding approach in image segmentation has be evaluated on three standard test images. These well-known images are Hunter, Lena and Peppers respectively, shown in Fig. 2(a)-(c). The test images are with size 512×512. Fig. 2(d)-(f) show the histograms of the three test images. In experiments, parameters of the proposed image thresholding approach based on cell-like P systems are given as follows:

(i) The used cell-like P system includes three evolution membranes ($m = 3$), where the number of objects contained in each evolution membrane is $n = 50$, and the maximum execution step number is $S_{max} = 100$;

(ii) In the used mutation rule (15), the scaling factor is set to be $F = 0.35$. In the used crossover rule (16), the crossover rate is set to be $CR = 0.2$.

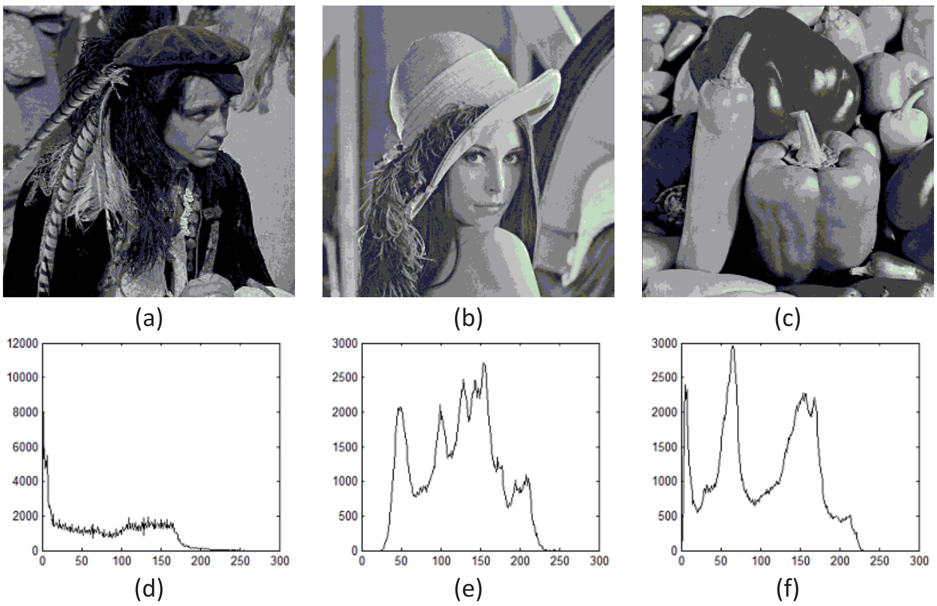

Fig. 2. Three test images ((a) Hunter; (b) Lena; (c) Peppers) and their histograms ((d) Hunter; (e) Lena; (f) Peppers).

In order to illustrate segmentation performance of the proposed three-level thresholding approach, its segmentation results are compared with the results obtained by PSO-based and GA-based approaches respectively. For the PSO-based approach, basic position-velocity model is employed and its parameters are set: population size $NP = 30$, maximum generation number $G_{max} = 100$,

$c_1 = c_2 = 1.0$, and w linearly varies from 0.9 to 0.4. For the GA-based approach, its parameters are given: population size $NP = 30$, crossover probability $P_c = 0.6$, mutation rate $P_m = 0.01$ and maximum generation number $G_{max} = 100$.

Table 1 lists their optimal segmentation thresholds. Fig. 3 gives optimal three-level segmentation results on above three test images for the proposed three-level thresholding approach based on cell-like P systems (in short, P systems), PSO-based approach (in short, PSO), GA-based approach (in short, GA), respectively. From the Fig. 3, we can see that results of the proposed three-level thresholding approach based on cell-like P systems is slightly better than that of PSO-based approach but evidently outperforms that of GA-based approach. This illustrates the applicability of the proposed approach for three-level thesholding.

Table 1. The optimal thresholds obtained by different methods.

Approaches	Lunter	Lena	Peppers
P systems	86, 179	98, 165	76, 152
PSO	82, 183	99, 166	80, 145
GA	73, 179	103, 168	85, 151

In order to investigate the efficiency, all approaches are compared based on the average CPU time (in seconds) taken to converge the solution. Comparison results of all methods given in Table 2. Form Table 2, it is clear that the proposed three-level thresholding approach based on cell-like P systems has fast convergence compared with PSO-based and GA-based approaches. The results demonstrate that the proposed three-level thresholding approach based on cell-like P systems is more efficient and effective that other approaches for three-level thresholding.

Table 2. Comparison of CPU time (in seconds) for different methods.

Approaches	Lunter	Lena	Peppers
P systems	7.975	7.641	7.012
PSO	9.521	9.136	9.849
GA	11.973	11.654	12.117

5 Conclusion

In this paper, we have presented a fast three-level thresholding approach based on cell-like P systems, which employed the total fuzzy entropy as the evaluation criterion. In order to effectively exploit the optimal segmentation thresholds, a special membrane structure with three layers was designed, which allows multiple membranes to co-evolve the objects of the system, and an improved evolution

Fig. 3. Three-level thresholding images obtained by different methods. (a)-(c) P systems; (d)-(f) PSO; (g)-(i) GA.

operation of DE was used as evolution rules of these membranes. With the special membrane structure and mechanism of the cell-like P system, two best objects were used to guide the evolution of the objects: one was best object from the corresponding local store membrane and another was from global store membrane. This mechanism not only effectively accelerates the speed of convergence but also enhances the diversity of objects in the system. The proposed thresholding approach based on cell-like P systems has been tested on several standard images and were compared with GA-based and PSO-based approaches. The experimental results showed the proposed thresholding approach outperforms the other approaches in terms of the applicability and computation efficiency. Further works are to be carried out to feasibility of the proposed thresholding approach for various types of image processing applications.

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